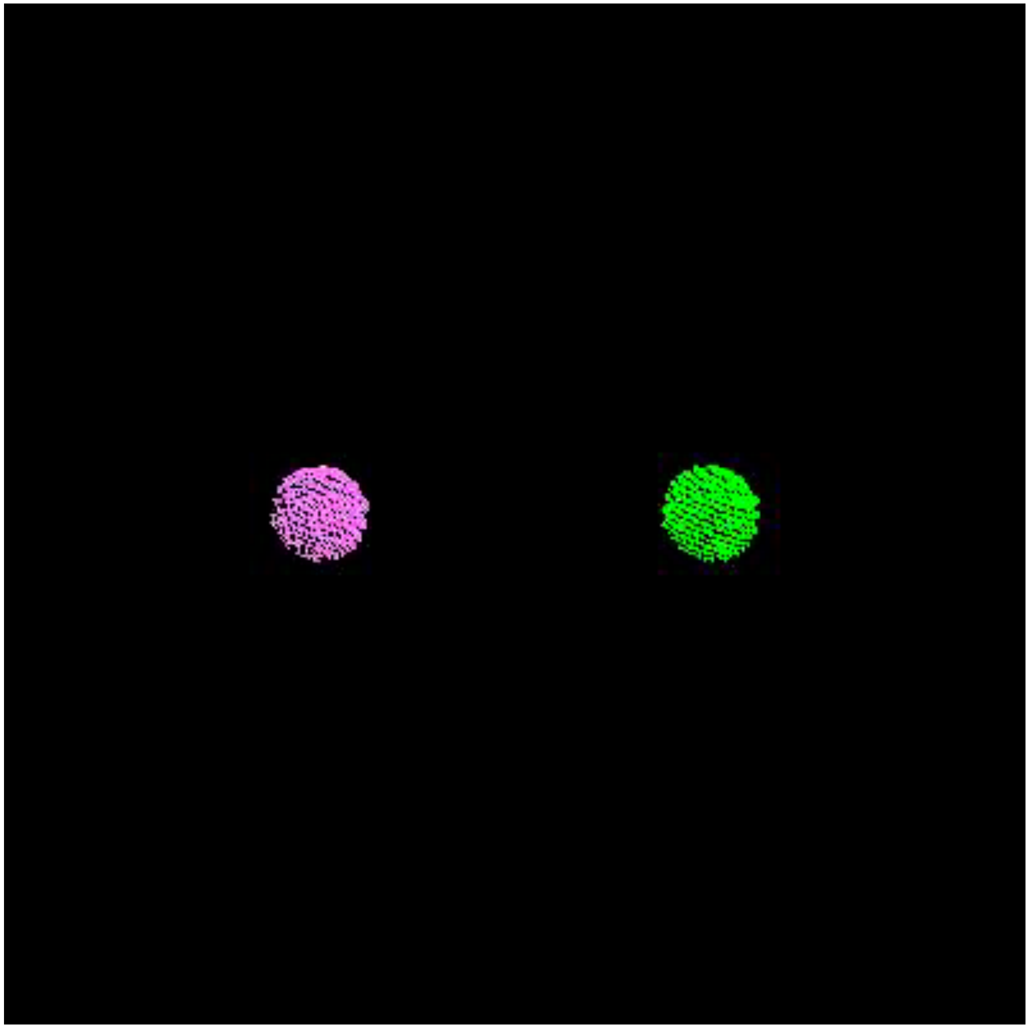
N-BODY COLLISIONS

Derek C. Richardson
University of Maryland

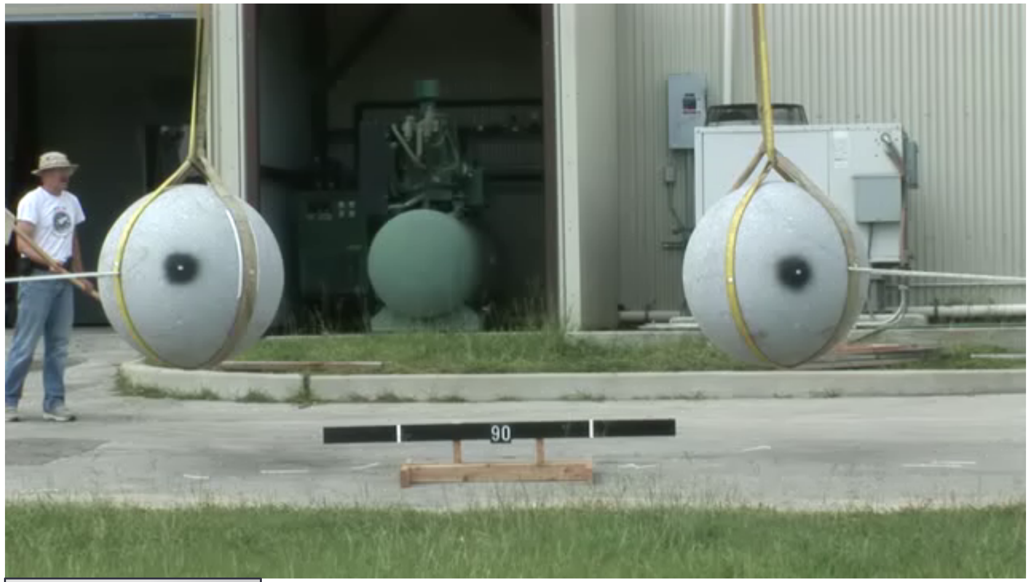
Simulating Collisions



Leinhardt+00

- Collisions are integral to planet formation simulations.
- In most scenarios, planetesimals, then planets, build up through merger of smaller pieces.

Simulating Collisions



Durda+11

- Because of their stochastic and impulsive nature, collisions are a challenge to include in simulations.

Simulating Collisions: Strategies

- Analytic/statistical/Monte Carlo: will not discuss here.
- Direct: gravity equations of motion integrated explicitly…

$$\ddot{\mathbf{r}}_i = -\sum_{j \neq i} \frac{G m_j (\mathbf{r}_i - \mathbf{r}_j)}{\left|\mathbf{r}_i - \mathbf{r}_j\right|^3}$$

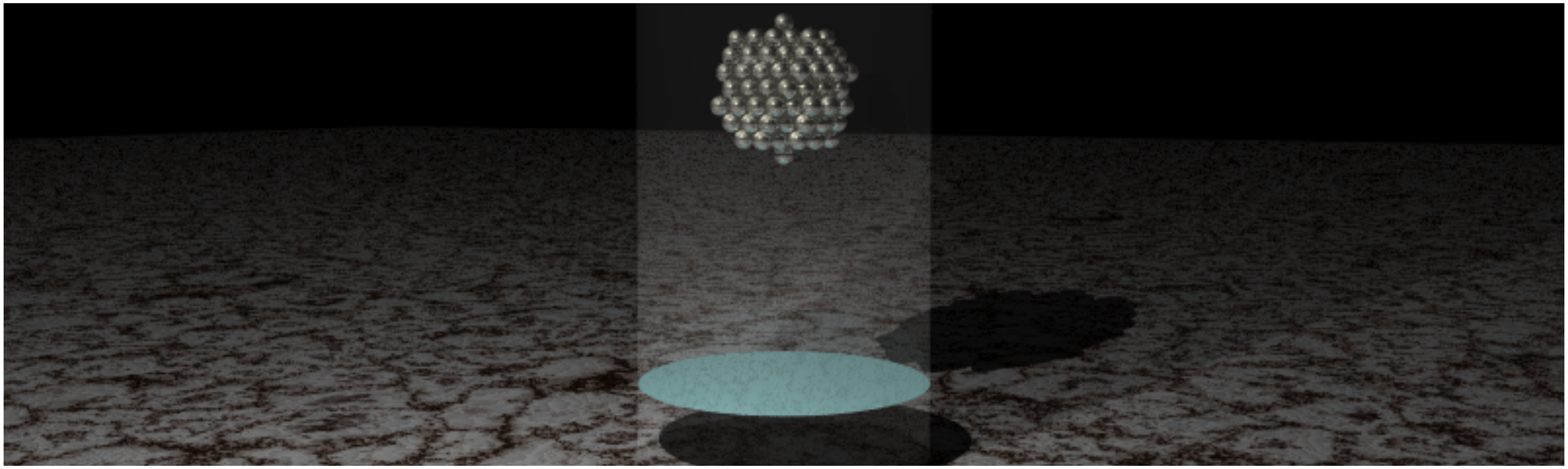
m = point mass
r = vector position

…with collision condition,

Separation → $$\left|\mathbf{r}_i - \mathbf{r}_j\right| \leq s_i + s_j.$$ ← Sum of radii

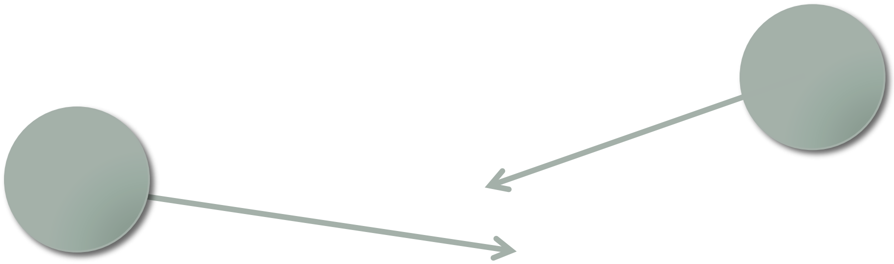
Richardson+00

Hard-Sphere Discrete Element Method



- HSDEM (billiard-ball physics): idealized, point-contact, zero-duration collisions.
- Predict collision events in advance, or detect (unphysical) overlap, then fix. Sometimes both!
- Appropriate in low-density regimes where time between collisions is long and multiple contacts rare/unimportant.

HSDEM: Collision Prediction



$$\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$$
$$\mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1$$

Collision condition at time *t*:

$$v^2 t^2 + 2(\mathbf{r}\cdot\mathbf{v})t + r^2 = (s_1 + s_2)^2.$$

Solve for *t* (take smallest positive root):

$$t = \frac{-(\mathbf{r}\cdot\mathbf{v}) \pm \sqrt{(\mathbf{r}\cdot\mathbf{v})^2 - \left[r^2 - (s_1 + s_2)^2\right]v^2}}{v^2}.$$

Richardson+00

HSDEM: Collision Resolution

Richardson 94

Post-collision velocities and spins:

$$\mathbf{v}_1' = \mathbf{v}_1 + \frac{m_2}{M}\left[(1+\varepsilon_n)\mathbf{u}_n + \beta(1-\varepsilon_t)\mathbf{u}_t\right],$$

$$\mathbf{v}_2' = \mathbf{v}_2 - \frac{m_1}{M}\left[(1+\varepsilon_n)\mathbf{u}_n + \beta(1-\varepsilon_t)\mathbf{u}_t\right],$$

$$\vec{\omega}_1' = \vec{\omega}_1 + \beta\frac{\mu}{I_1}(1-\varepsilon_t)(\mathbf{s}_1 \times \mathbf{u}),$$

$$\vec{\omega}_2' = \vec{\omega}_2 - \beta\frac{\mu}{I_2}(1-\varepsilon_t)(\mathbf{s}_2 \times \mathbf{u}),$$

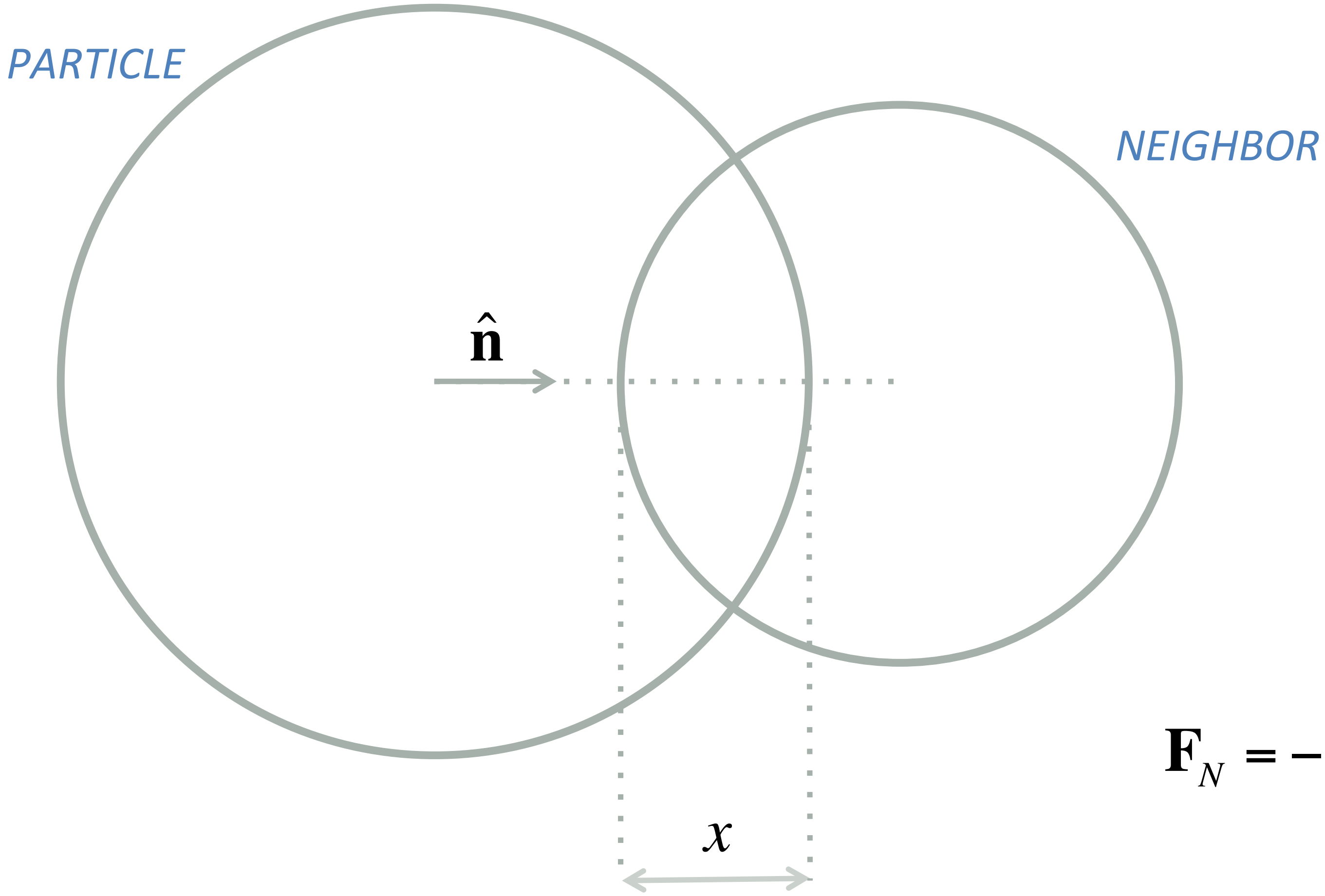
where:

$M = m_1 + m_2$, $\mu = m_1 m_2/M$, $\mathbf{u} = \mathbf{v} + \boldsymbol{\sigma}$, $\hat{\mathbf{n}} = \mathbf{r}/r$, $\mathbf{u}_n = (\mathbf{u}\bullet\hat{\mathbf{n}})\hat{\mathbf{n}}$, $\mathbf{u}_t = \mathbf{u} - \mathbf{u}_n$, $\mathbf{s}_1 = s_1\hat{\mathbf{n}}$, $\mathbf{s}_2 = -s_2\hat{\mathbf{n}}$, $\boldsymbol{\sigma}_i = \boldsymbol{\omega}_i \times \mathbf{s}_i$, $\boldsymbol{\sigma} = \boldsymbol{\sigma}_2 - \boldsymbol{\sigma}_1$, $\beta = 2/7$ for spheres, and $I_i = (2/5)\, m_i R_i^2$.

Soft-Sphere Discrete Element Method

- Strategy: allow particles to overlap (deform) in order to simulate the contact forces that arise during collision.
- Advantages:
  - Improved realism—multiple persistent contacts, true friction forces.
  - Adjustable parameters—rigidity (sound speed), friction.
  - Parallelizability—SSDEM forces can be computed in parallel.
- Disadvantages:
  - Need smaller timesteps (depends on rigidity).
  - Need more memory per particle (for tracking contact histories).
- Note: both HSDEM and SSDEM require fast search for particles neighbors ➔ tree code (and parallelism).

SSDEM: Normal Restoring Force



$$\mathbf{F}_N = -(k_n x)\hat{\mathbf{n}} + C_n \mathbf{u}_n .$$

SSDEM: Summary of Equations

Schwartz+12
(*Granular Matter* **14**, 363)

Force

Restoring force

Plastic friction forces

$$\mathbf{F}_1 = -(k_n x)\hat{\mathbf{n}} + C_n \mathbf{u}_n + \min\left\{\mu_s \left|\mathbf{F}_n\right| \hat{\mathbf{S}};\ k_t \mathbf{S} + C_t \left|\mathbf{u}_t\right| \hat{\mathbf{t}}\right\}.$$

Static friction force

Tangential friction torque

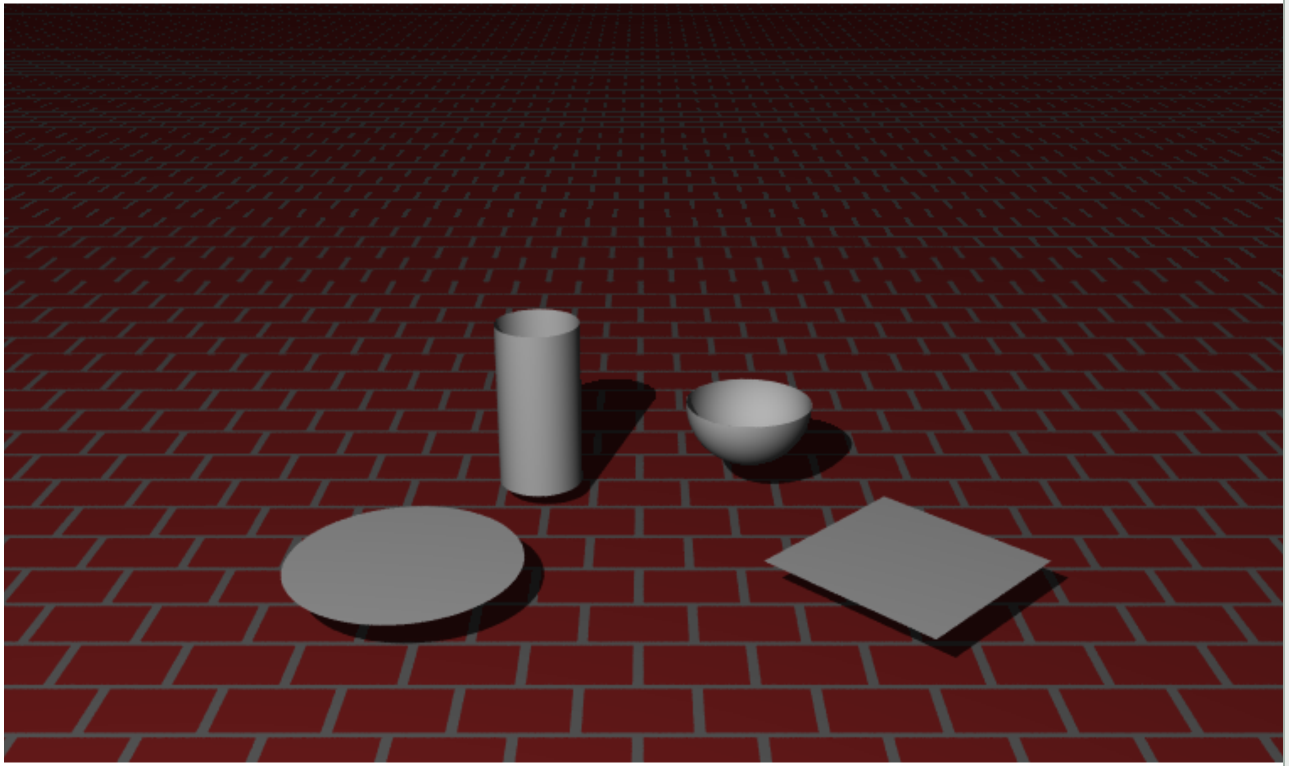
Rolling friction

Twisting friction

$$\mathbf{M}_1 = -l_1 \left[\min\left\{\mu_s \left|\mathbf{F}_n\right| \hat{\mathbf{S}};\ k_t \mathbf{S} + C_t \left|\mathbf{u}_t\right| \hat{\mathbf{t}}\right\}\right] \times \hat{\mathbf{n}} + \mu_r \left|\mathbf{F}_n\right| l_1 \left[\frac{\mathbf{v}_{\mathrm{rot}} \times \hat{\mathbf{n}}}{\left|\mathbf{v}_{\mathrm{rot}}\right|}\right] + \mu_t \left|\mathbf{F}_n\right| r_c \left[\frac{(\vec{\omega}_2 - \vec{\omega}_1)\cdot\hat{\mathbf{n}}}{\left|(\vec{\omega}_2 - \vec{\omega}_1)\cdot\hat{\mathbf{n}}\right|}\right]\hat{\mathbf{n}}.$$

Torque

Boundary Conditions



Ray-traced with POV-Ray

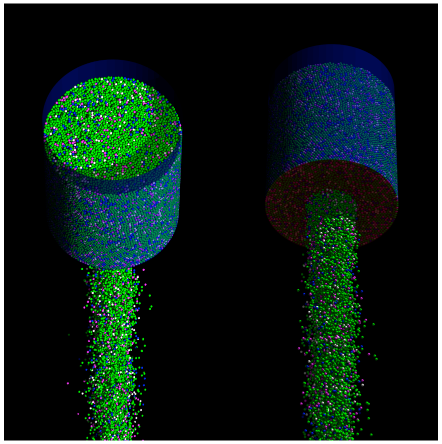
Richardson+11

```
wall type plane
  transparency 1

wall type disk
  origin -1 0 0.2
  orient 0 0 1
  radius 0.5

wall type cylinder-finite
  origin -0.5 1 0.5
  radius 0.2
  length 0.8

wall type shell
  origin 0.5 1 0.5
  radius 0.3
  open-angle 90

wall type rectangle
  origin 0.5 0 0.2
  vertex1 -0.6 0.6 0
  vertex2 0.6 0.6 0
```

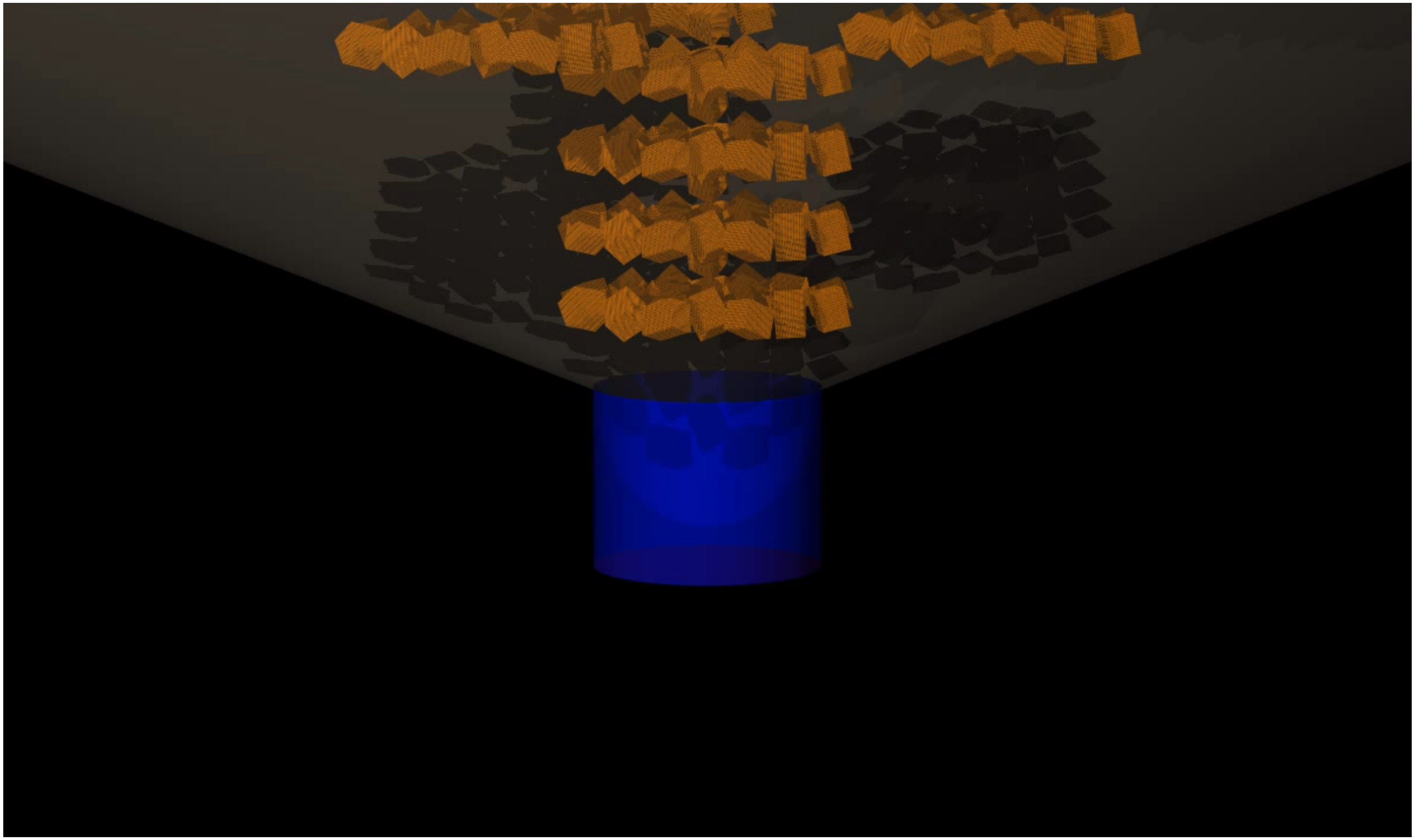
Granular Hopper Silos

- Validate approach by comparing against well-verified empirical results.

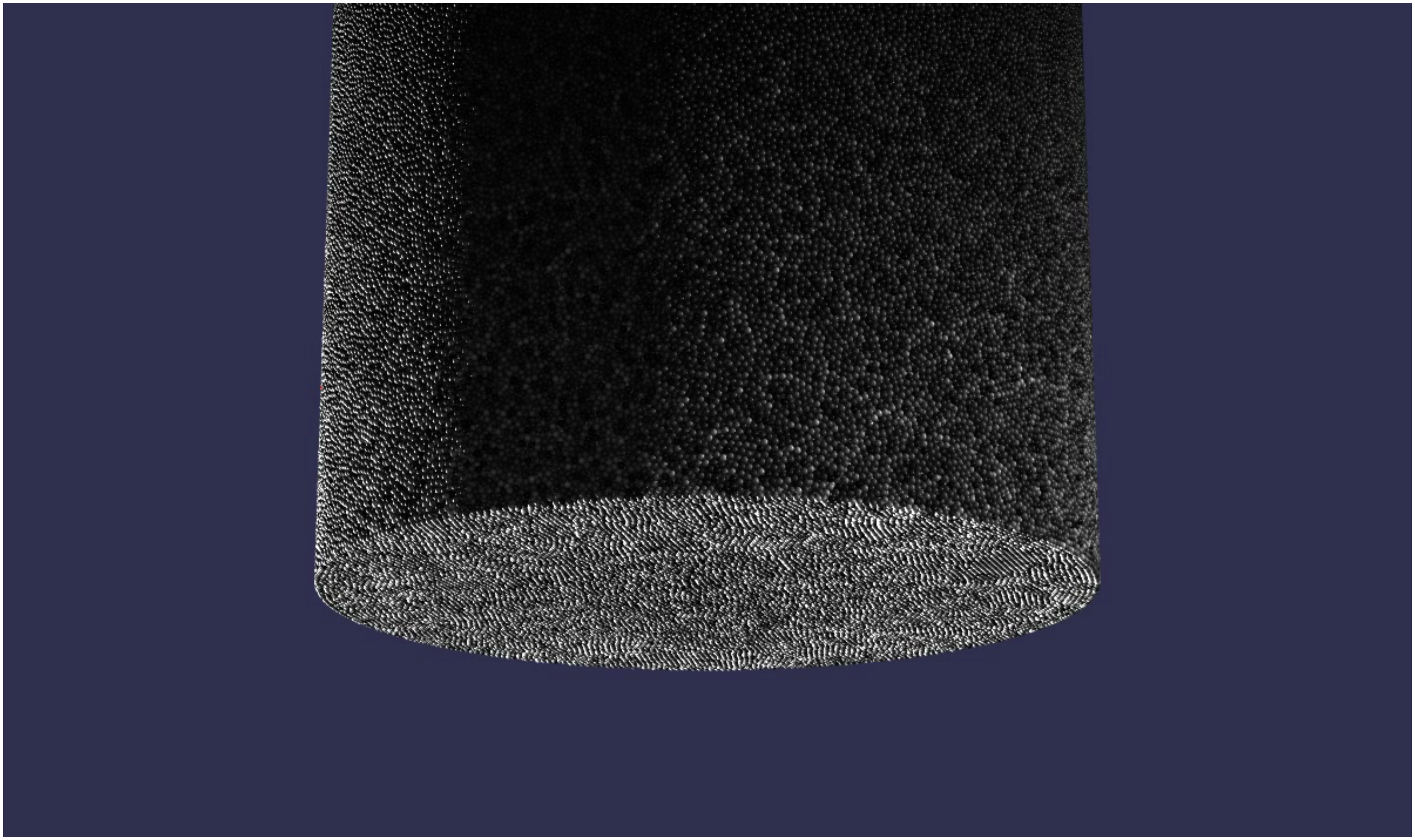
(E.g., flow rates from Beverloo+61.)



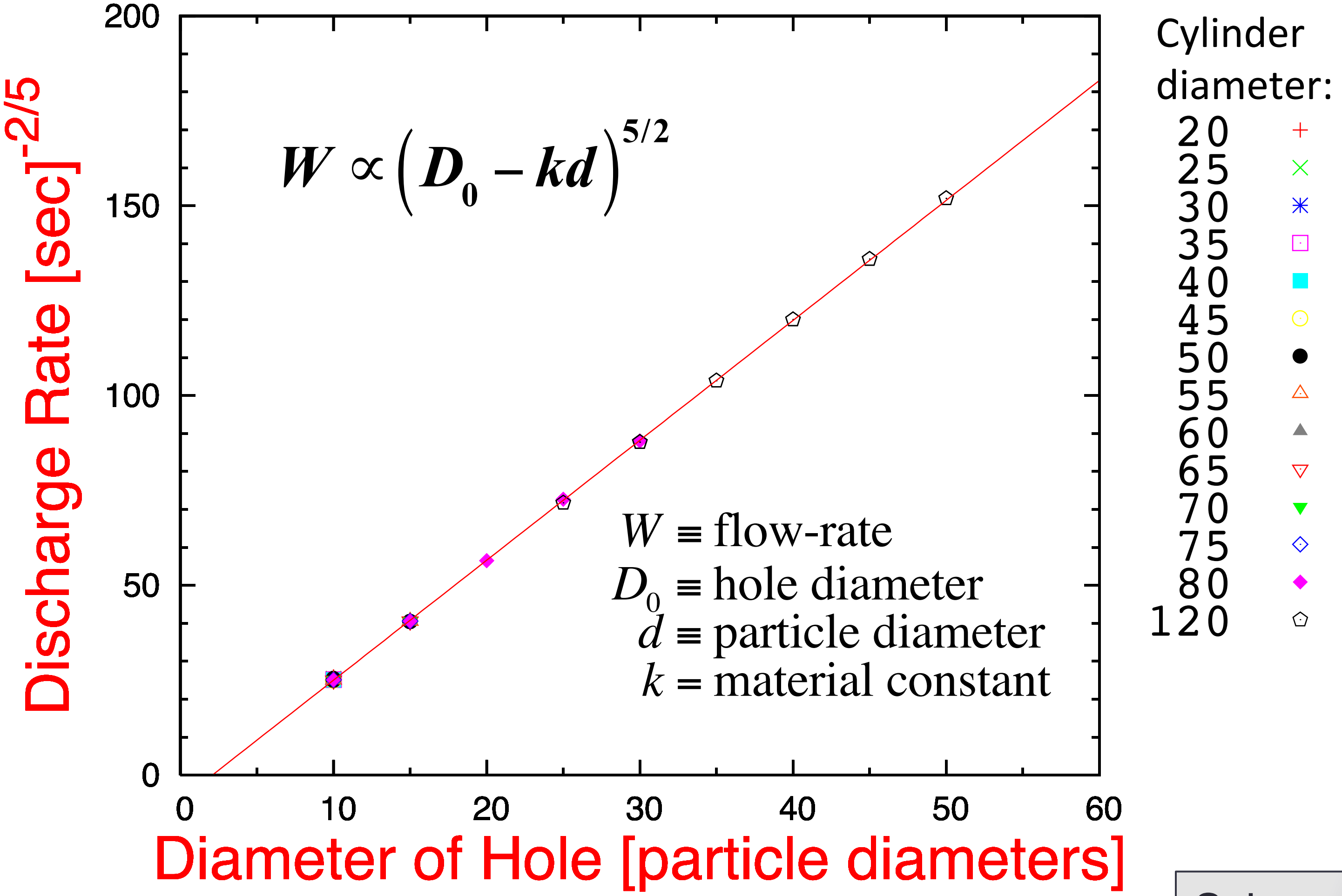
SSDEM Test: Hopper ($N = 1.5\times10^6$)



Hopper: Force Networks



Hole Size/Flow-rate Correlation



$$W \propto \left(D_0 - kd\right)^{5/2}$$

$W \equiv$ flow-rate

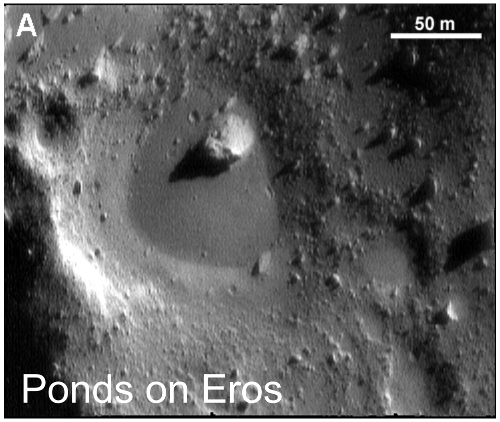
$D_0 \equiv$ hole diameter

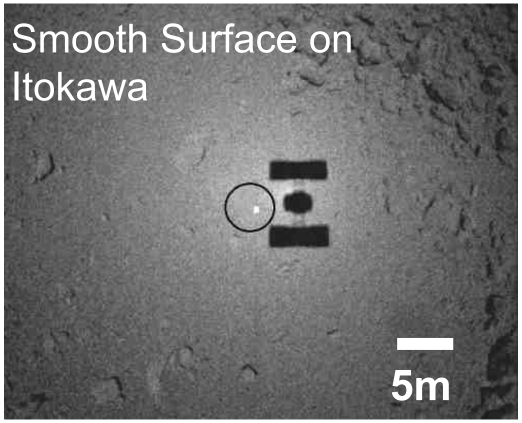
$d \equiv$ particle diameter

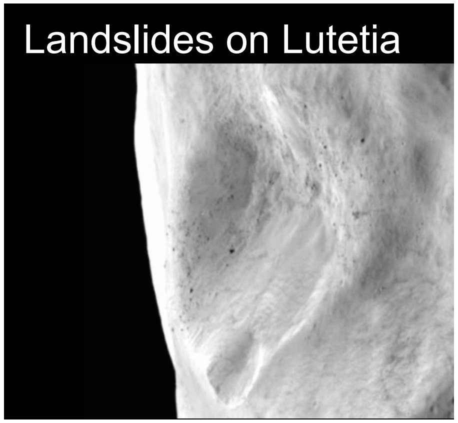
$k =$ material constant

Schwartz+12

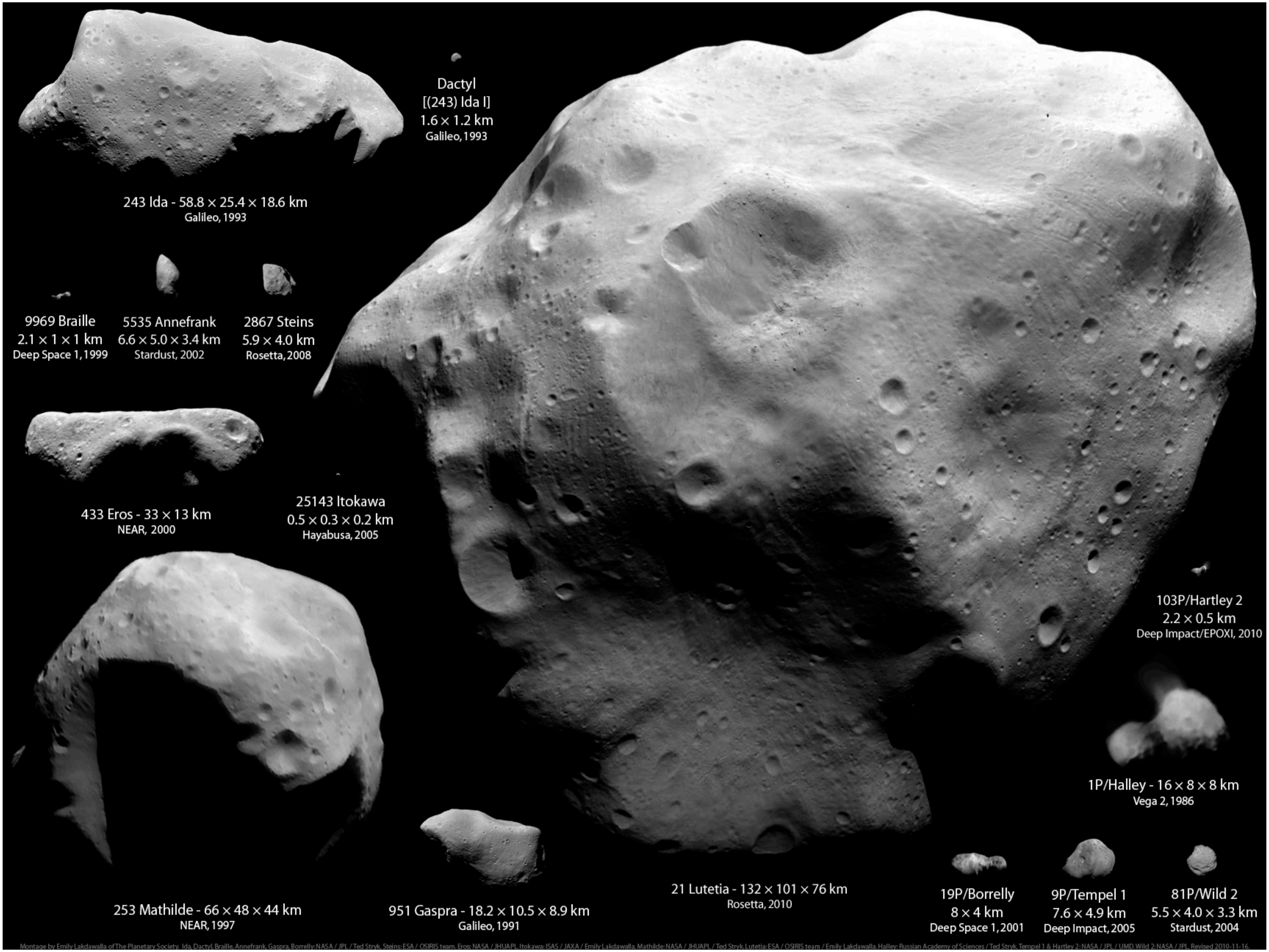
Why investigate granular material?



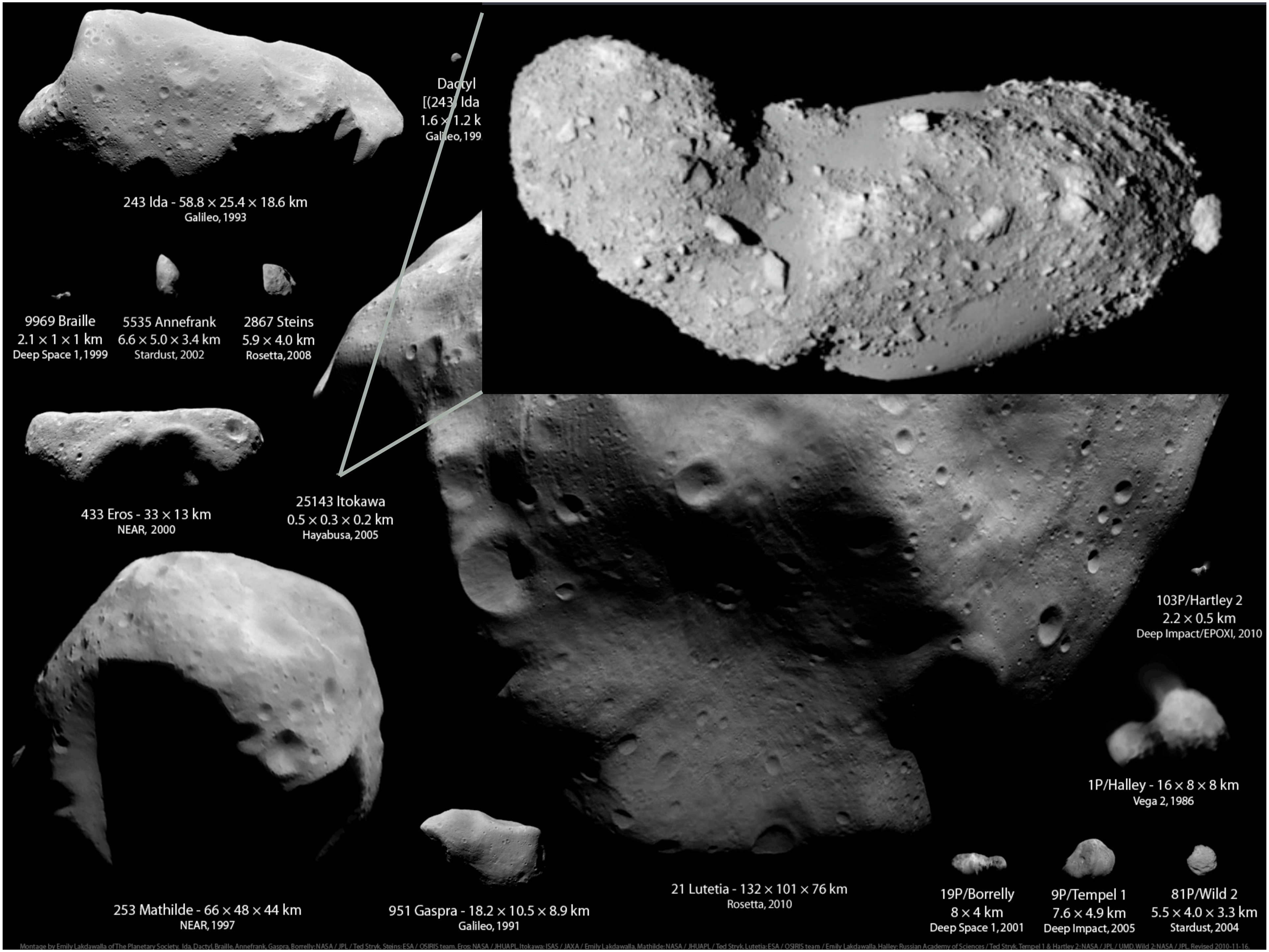




- Surfaces of planets and small bodies in our solar system are often covered by a layer of granular material.
- Understanding dynamics of granular material under varying gravitational conditions is important in order to:
  1. Interpret the surface geology of small bodies.
  2. Aid in the design of a successful sampling device or lander.



Montage by Emily Lakdawalla of The Planetary Society. Ida, Dactyl, Braille, Annefrank, Gaspra, Borrelly: NASA / JPL / Ted Stryk. Steins: ESA / OSIRIS team. Eros: NASA / JHUAPL. Itokawa: ISAS / JAXA / Emily Lakdawalla. Mathilde: NASA / JHUAPL / Ted Stryk. Lutetia: ESA / OSIRIS team / Emily Lakdawalla. Halley: Russian Academy of Sciences / Ted Stryk. Tempel 1 & Hartley 2: NASA / JPL / UMD. Wild 2: NASA / JPL. Revised 2010-11-16.



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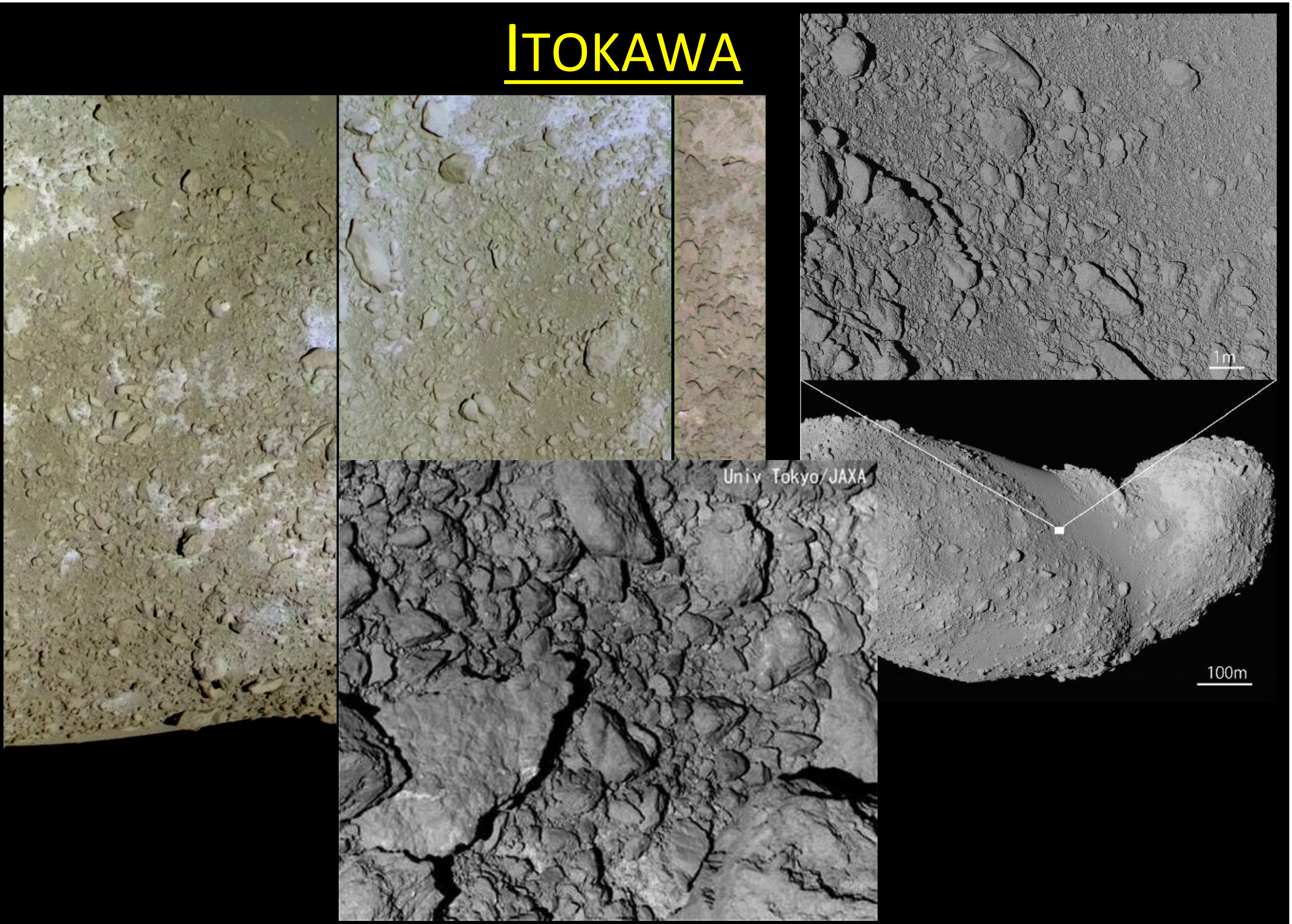
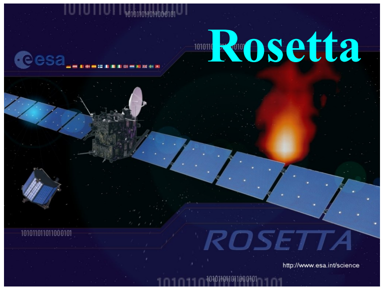
ITOKAWA



Upcoming Small Body Touchdowns

- **Hayabusa 2 (2014)**
  - Japanese Aerospace Exploration Agency (JAXA) funded mission to C-class asteroid.
- **OSIRIS-REx (2016)**
  - National Aeronautics and Space Administration (NASA) funded mission to B-class ssteroid.
- **MarcoPolo-R (2023 ?)**
  - European Space Agency (ESA) proposed mission to C-class asteroid.
- **Rosetta (arrive 2014)**
  - Active ESA mission to Mars-crossing comet.



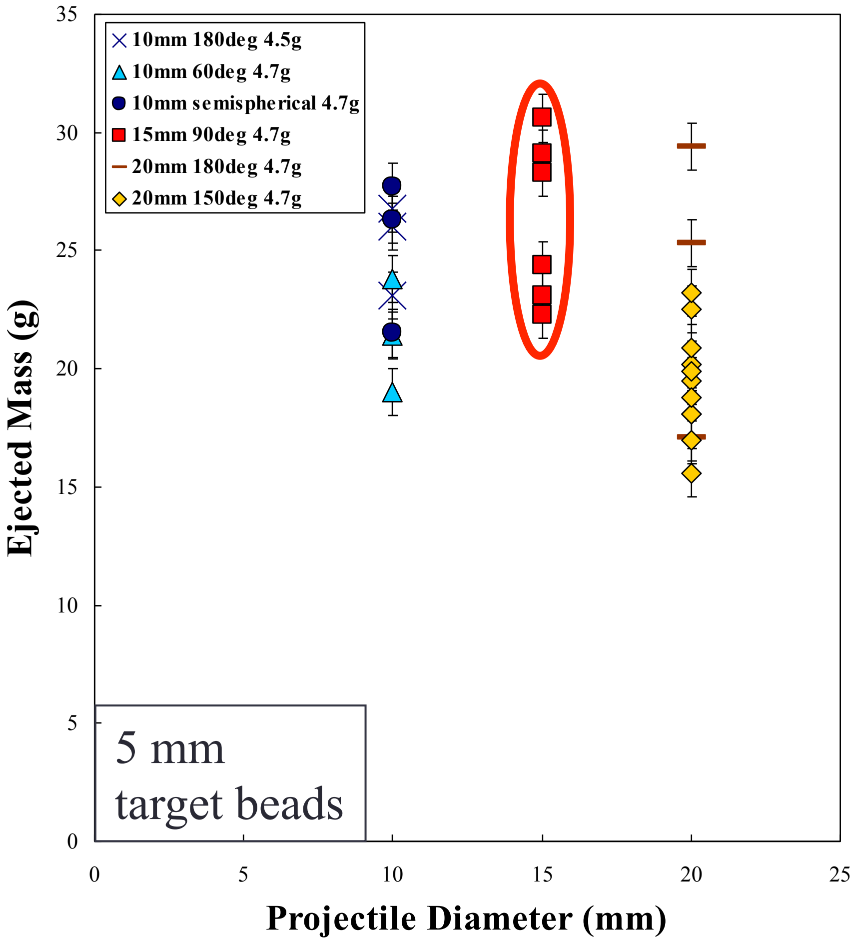
Hayabusa 2 Sampling Mechanism Experiment

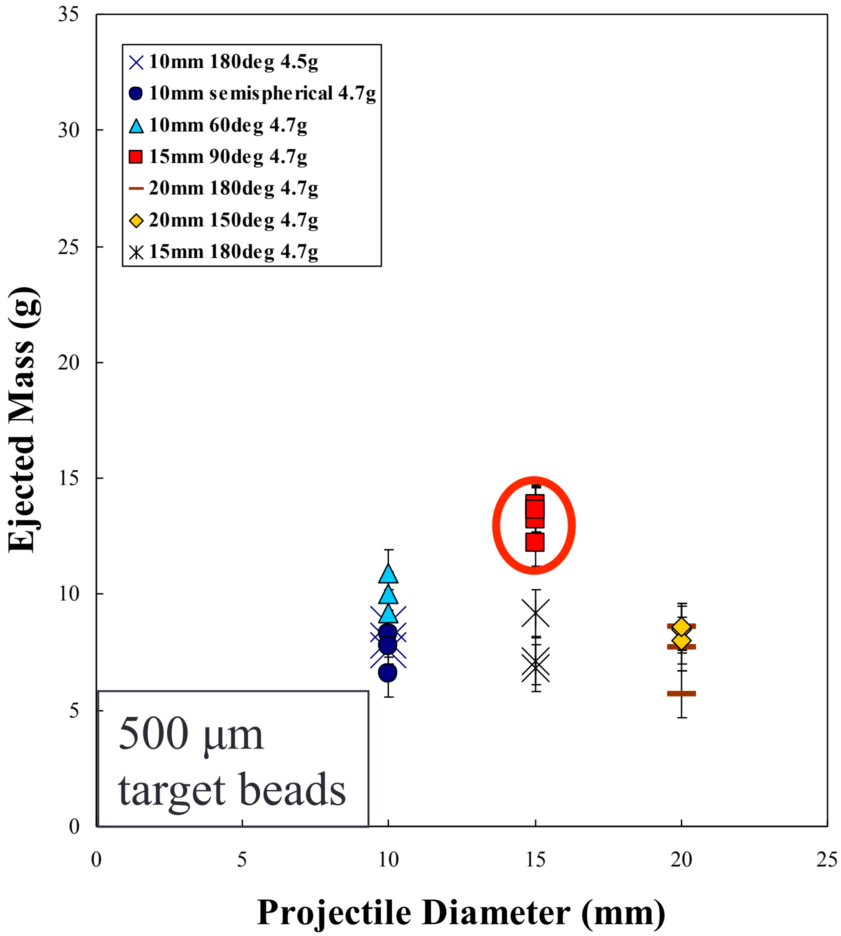
5 mm
target beads

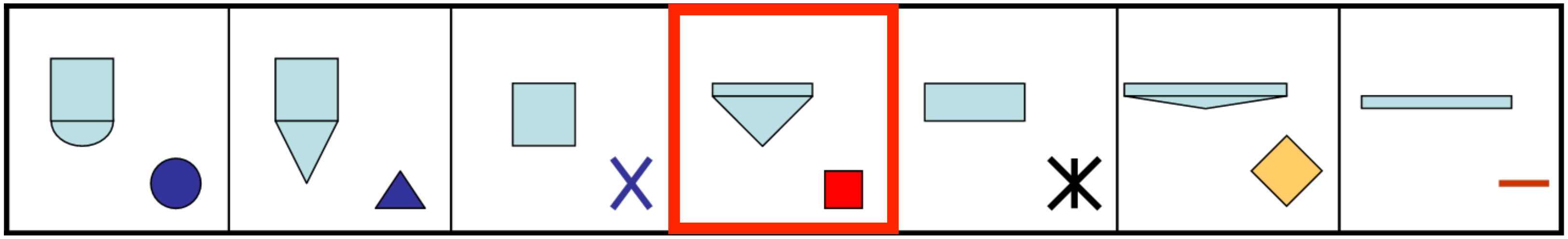
500 µm
target beads

Courtesy: Hajime
Yano, JAXA

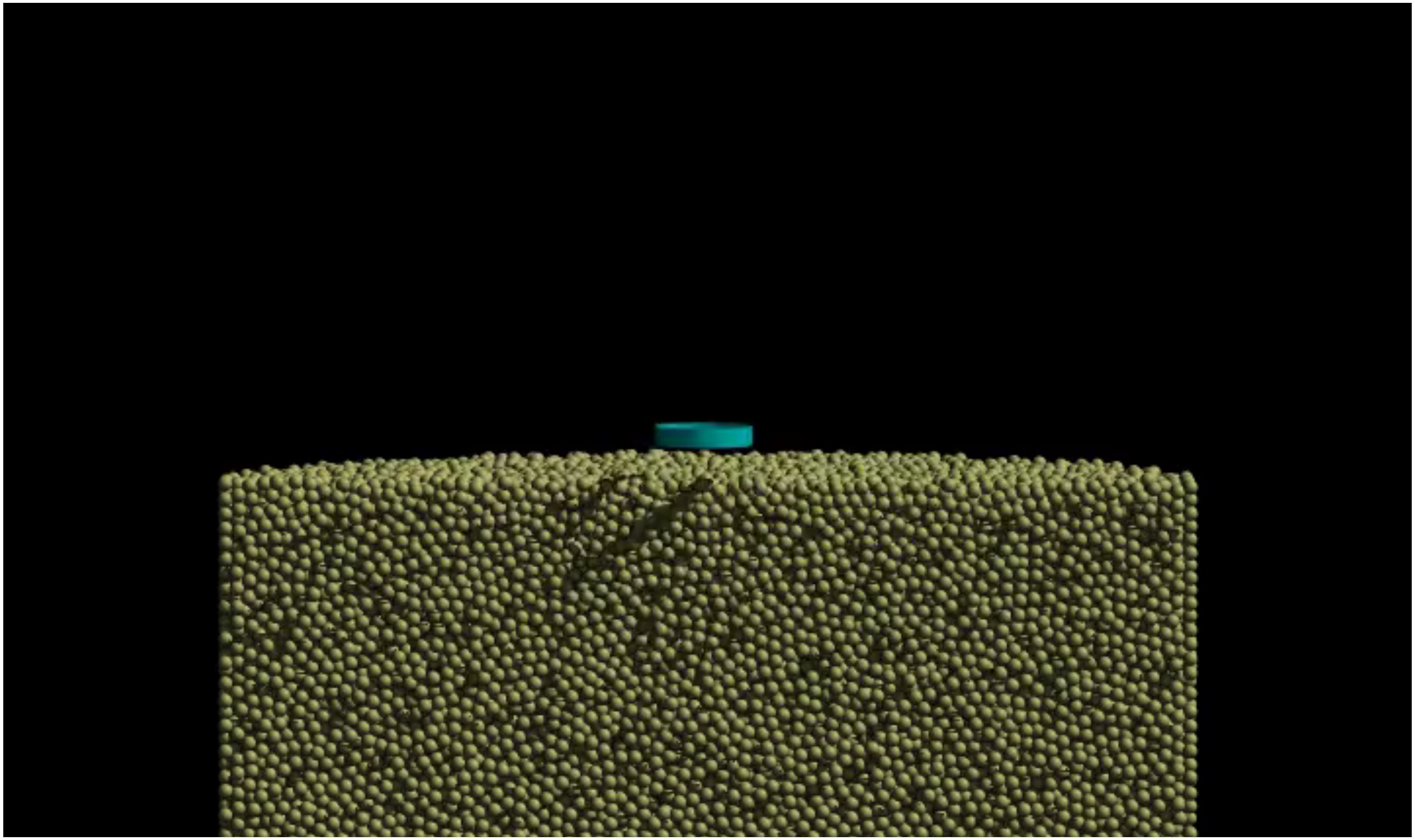
Experimental Results: Ejected Mass [11 m/s]





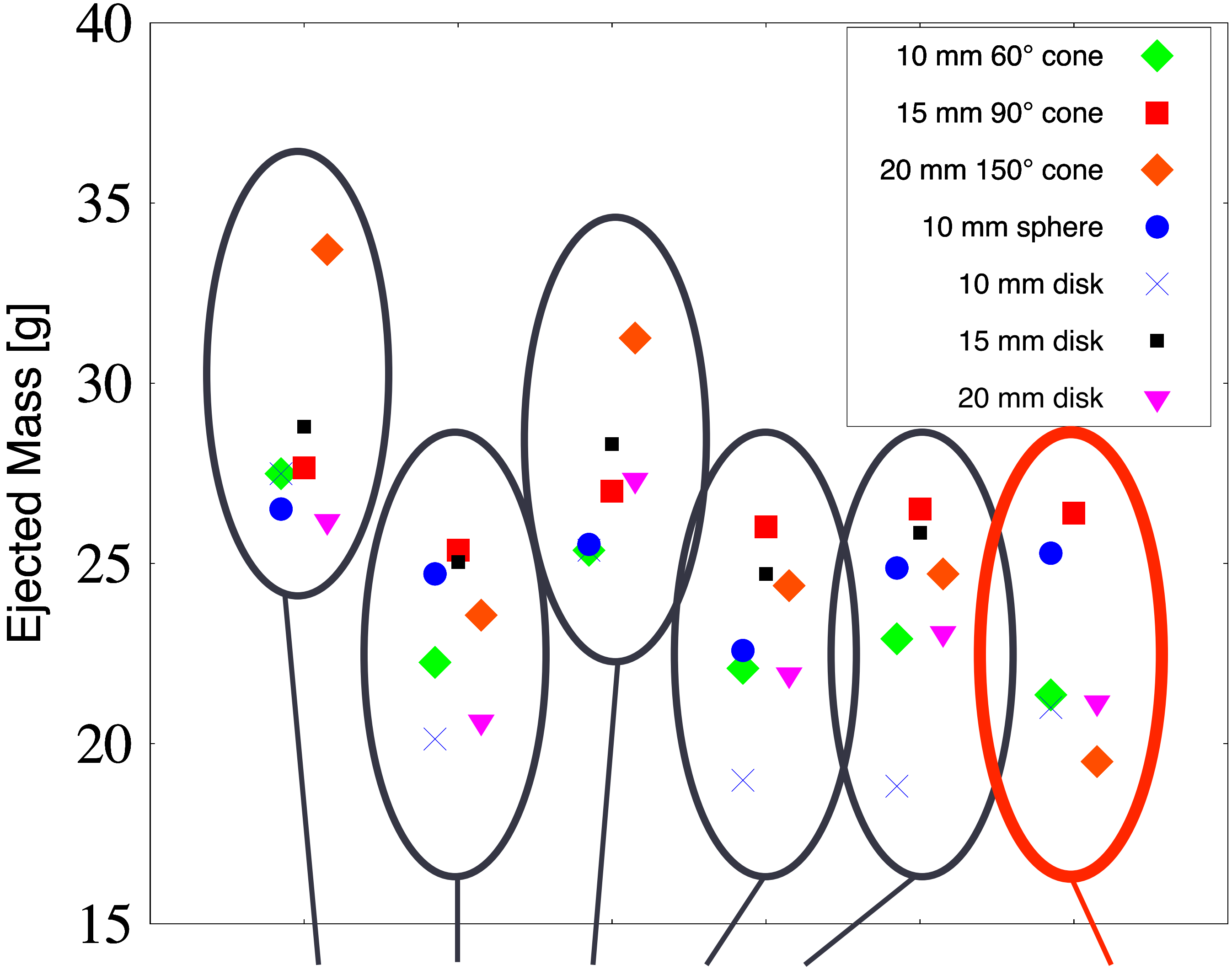


Hayabusa 2 Sampling Mechanism Simulation



Numerical Result: Ejected Mass [11 m/s]

(difference in target weight before and after)



5 mm
target beads

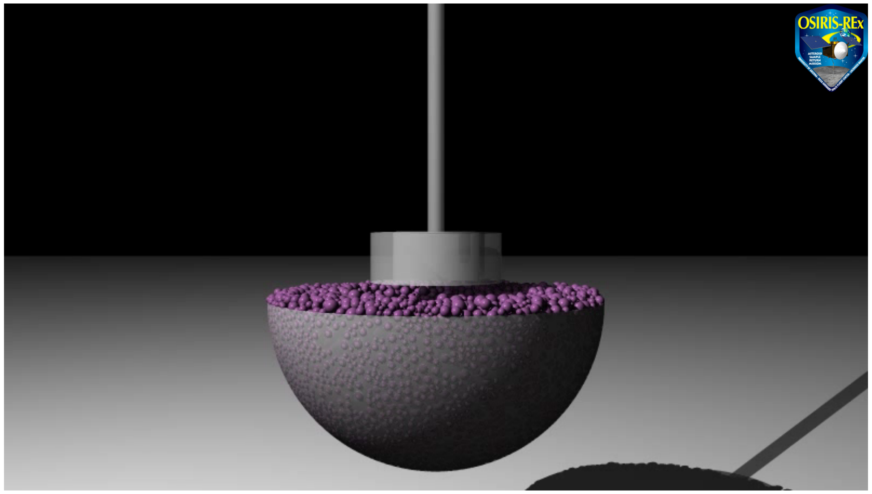
5 sets of simulations

Each has ϵ_n=0.95

Experiment

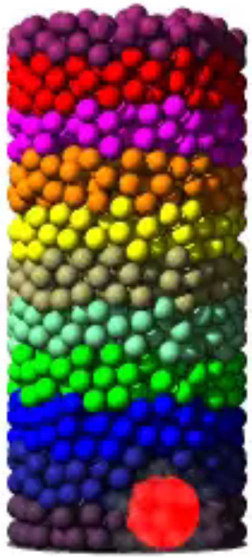
Schwartz+13
(in prep)

Some Ongoing Studies…



OSIRIS-REx
Compliance

Ronald Ballouz

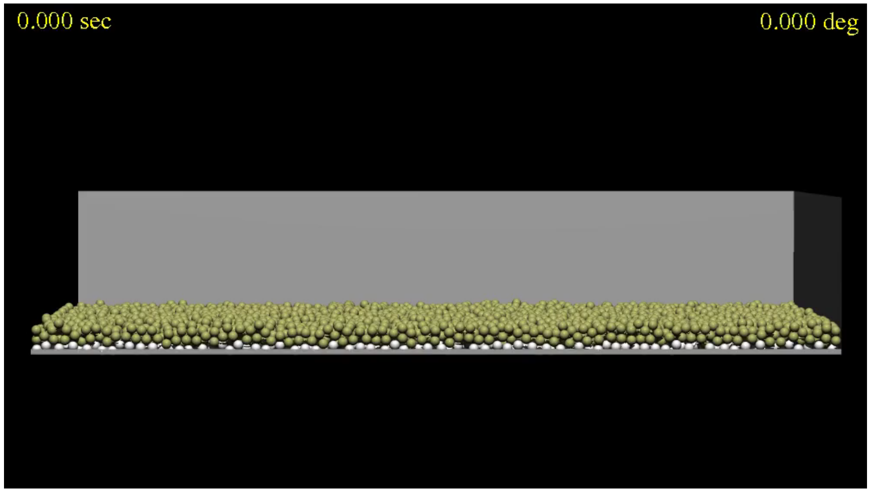


The Brazil Nut
Effect

Soko Matsumura

Granular
Avalanches

DCR



EXTRA SLIDES

Second-order Leapfrog

- Kick-drift-kick (KDK) scheme:

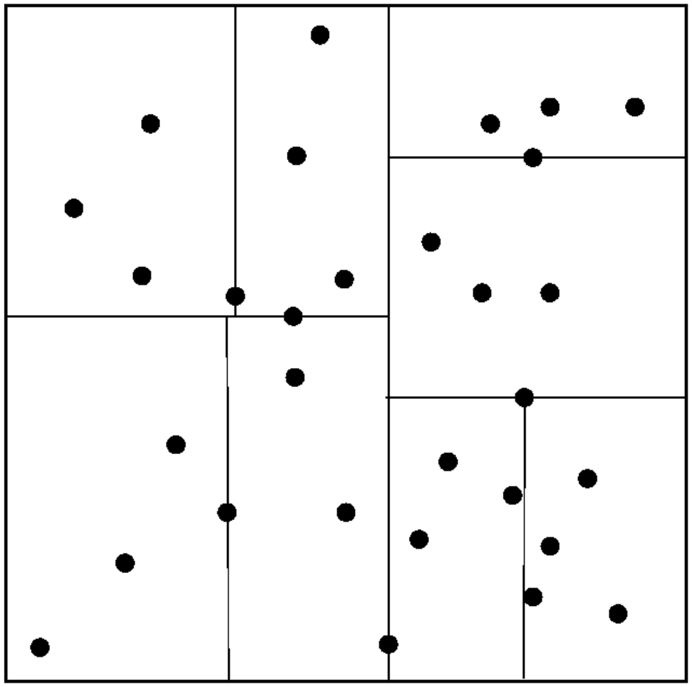
$$\begin{aligned}\dot{\mathbf{r}}_{i,n+1/2} &= \dot{\mathbf{r}}_{i,n} + (h/2)\ddot{\mathbf{r}}_{i,n} \quad \text{"kick"},\\ \mathbf{r}_{i,n+1} &= \mathbf{r}_{i,n} + h\dot{\mathbf{r}}_{i,n+1/2} \quad \text{"drift"},\\ \dot{\mathbf{r}}_{i,n+1} &= \dot{\mathbf{r}}_{i,n+1/2} + (h/2)\ddot{\mathbf{r}}_{i,n+1} \quad \text{"kick"},\end{aligned}$$

- Notice the drift is linear in the velocities—exploit this to search for collisions (HSDEM).

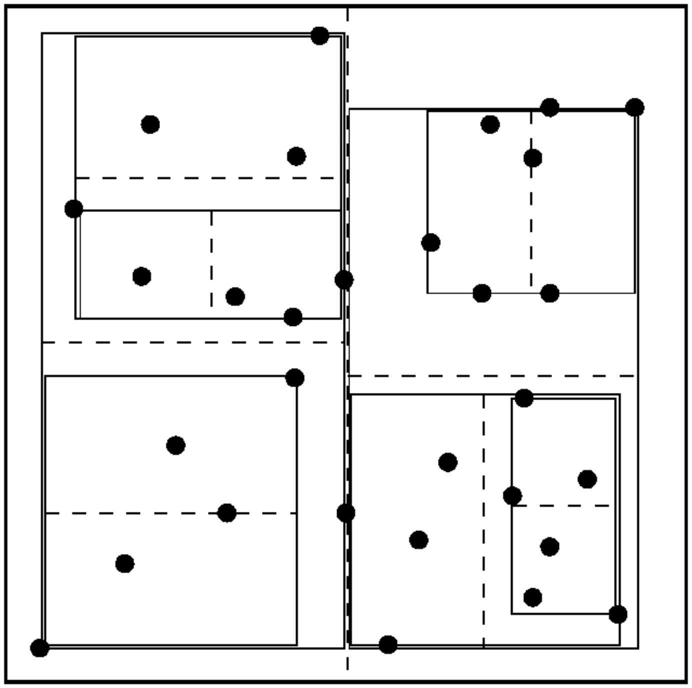
Some words about pkdgrav/gasoline.

- First developed at U Washington, this is a parallel, hierarchical gravity solver for problems ranging from cosmology to planetary science.
- “Parallel *k*-D Gravity code” = pkdgrav.
- Gasoline is pkdgrav with SPH enabled.
- Not released into the public domain (yet).
- If you’re interested in using it, see me!

Spatial Binary Tree



k-D Tree

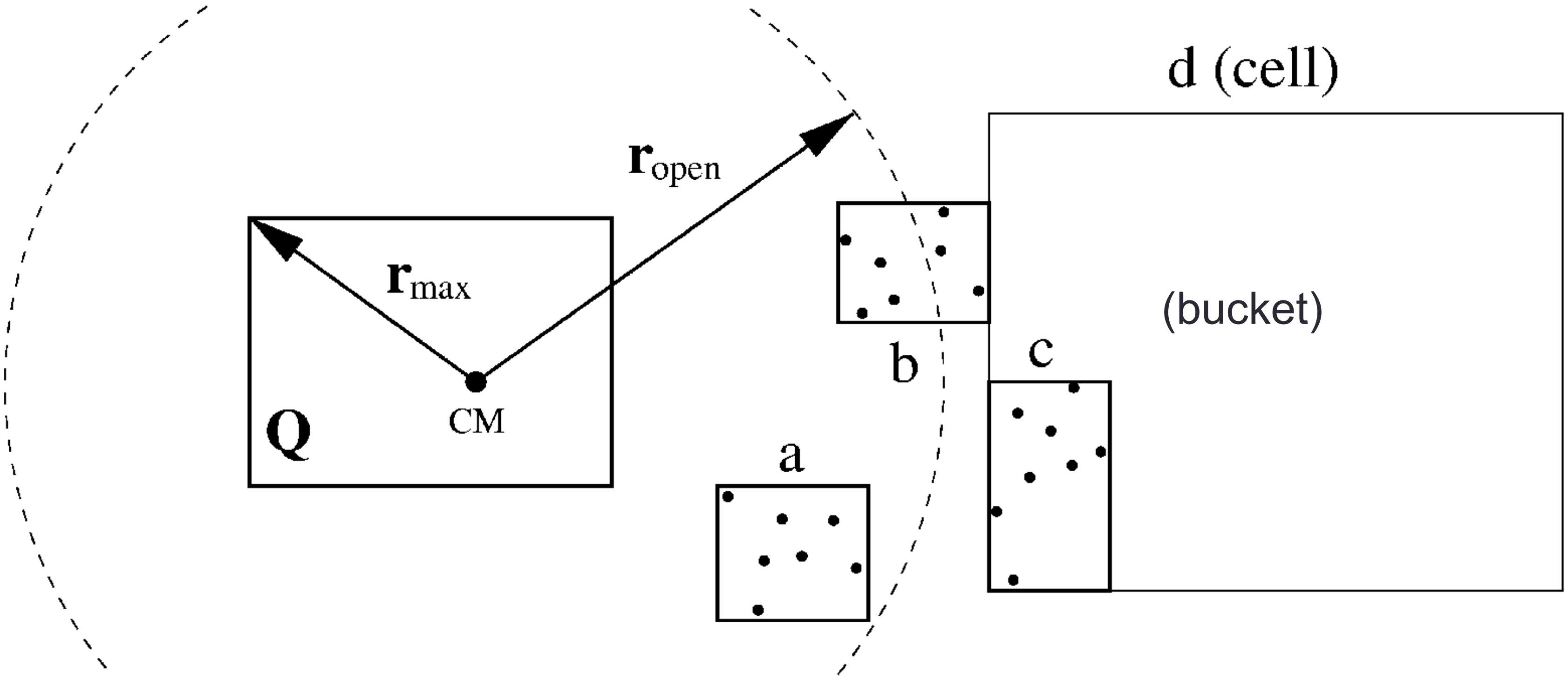


Spatial Binary Tree with Squeeze

Tree Walking

- Construct particle-particle and particle-cell interaction lists from top down for particles one bucket at a time.
- Define opening ball (based on *critical opening angle* θ) to test for cell-bucket intersection.
  - If bucket outside ball, apply multipole (c-list).
  - Otherwise open cell and test its children, etc., until leaves reached (which go on p-list).
- Nearby buckets have similar lists: amortize.

Tree Walking



Note multipole Q acceptable to all particles in cell d.

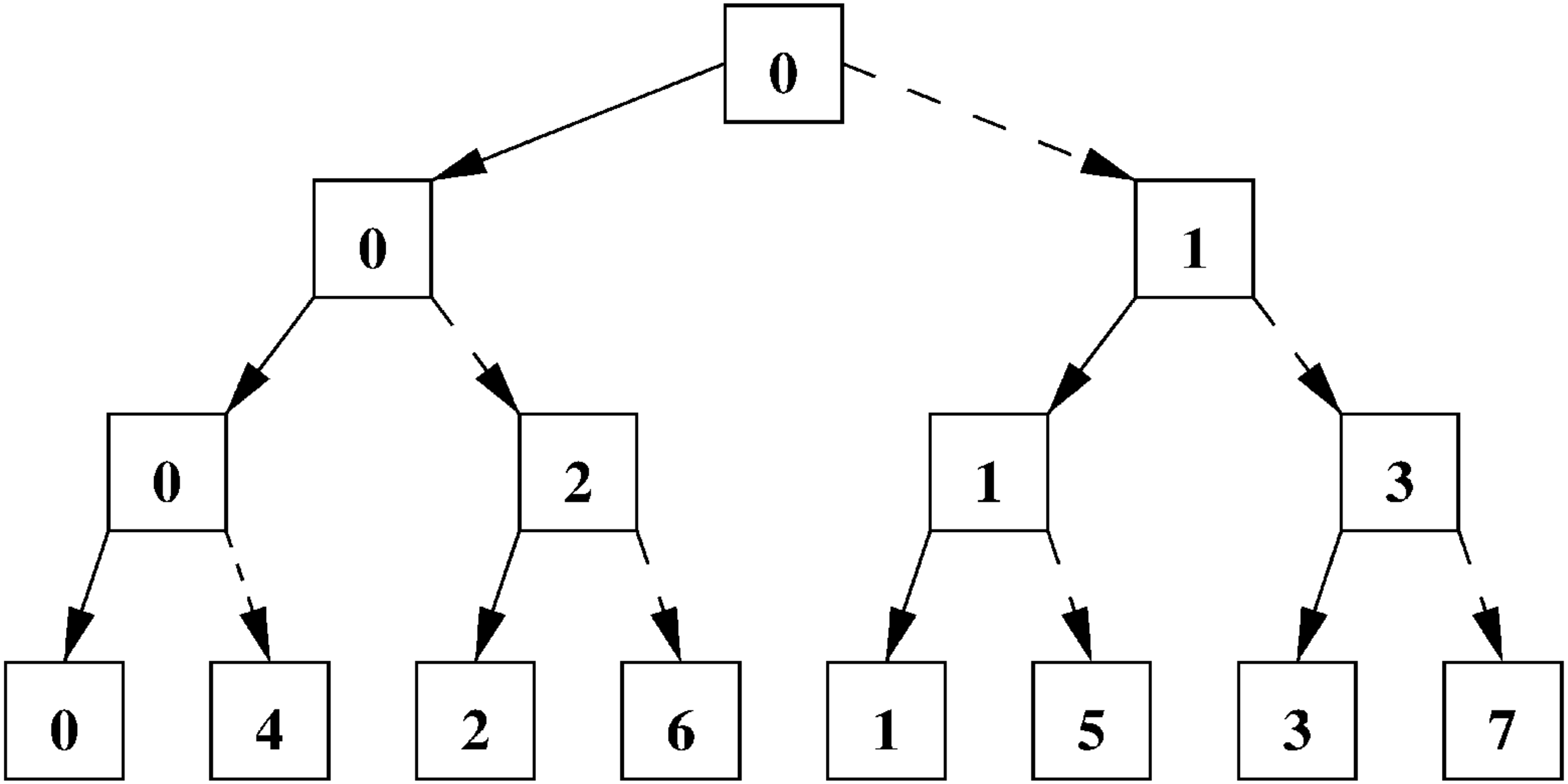
Other Issues

- Multipole expansion order.
  - Use hexadecapole (best bang for buck).
- Force softening (for cosmology).
  - Use spline-softened gravity kernel.
- Periodic boundary conditions.
  - Ewald summation technique available.
- Time steps.
  - Multistepping available (adaptive leapfrog).

Parallel Implementation

- Master layer (serial).
  - Controls overall flow of program.
- Processor Set Tree (PST) layer (parallel).
  - Assigns tasks to processors.
- Parallel *k*-D (PKD) layer (serial).
  - MIMD execution of tasks on each processor.
- Machine-dependent Layer (MDL, separate set of functions).
  - Interface to parallel primitives.

Domain Decomposition



Binary tree balanced by work factors. Nodes construct local trees.