Kobe 2013 Workshop
August 7-8

Critical Importance of Erosive Collisions in Collisional Cascade.

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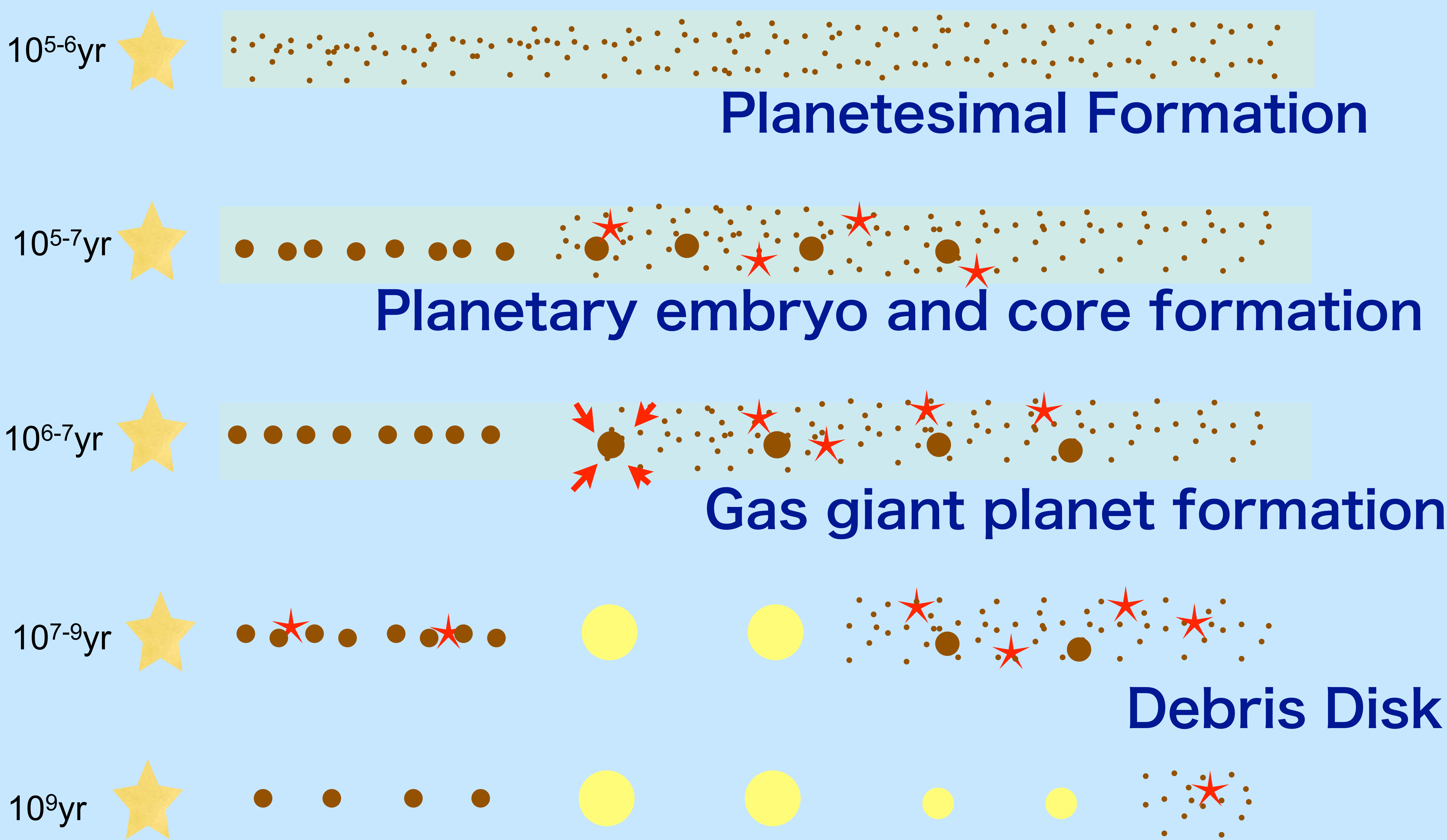
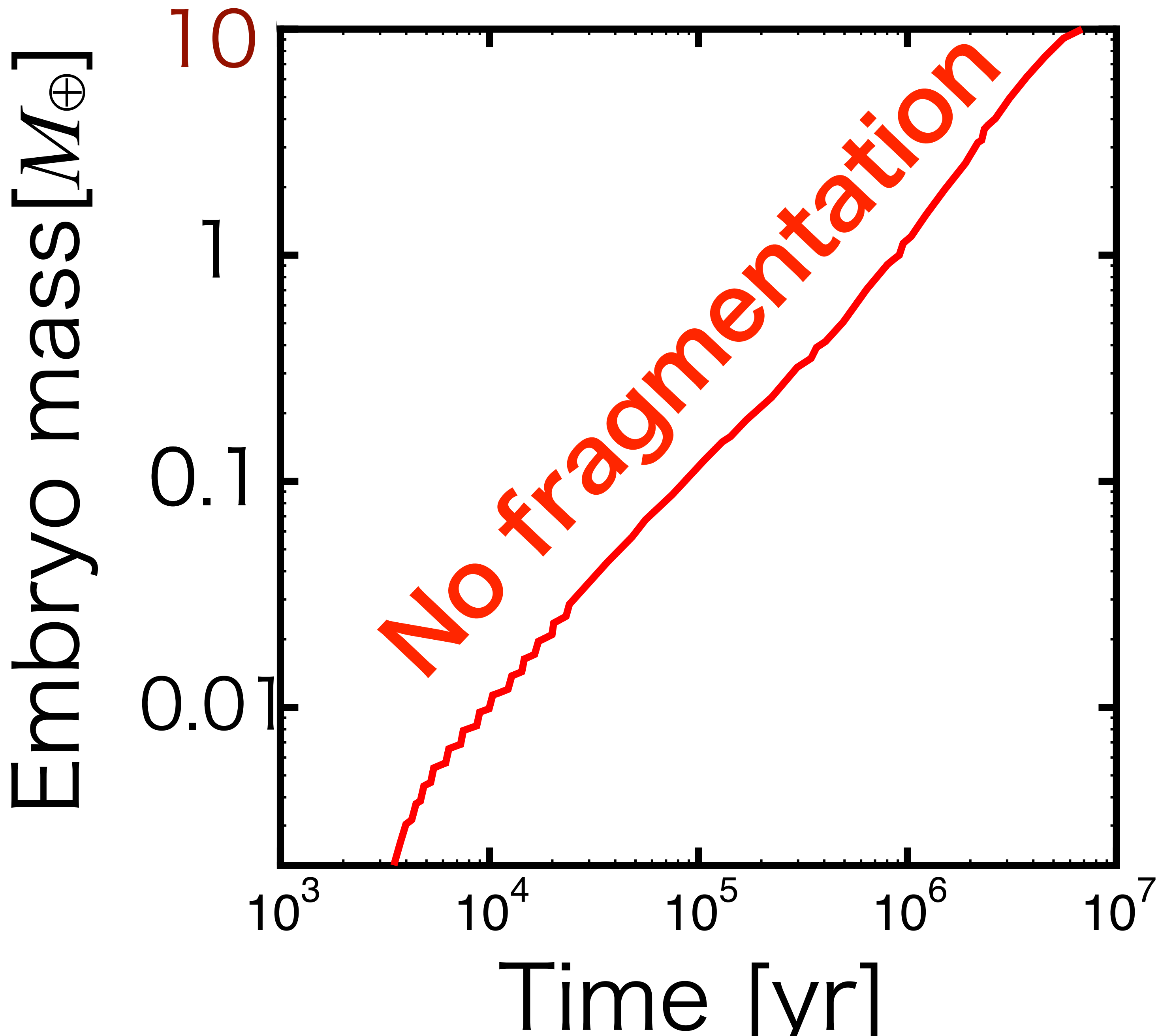
- Collisional fragmentation in planetary accretion
- Collisional cascade
- Importance of erosive (cratering) collisions

(See; Kobayashi & Tanaka 2010)

Planet Formation Model



Embryo Growth

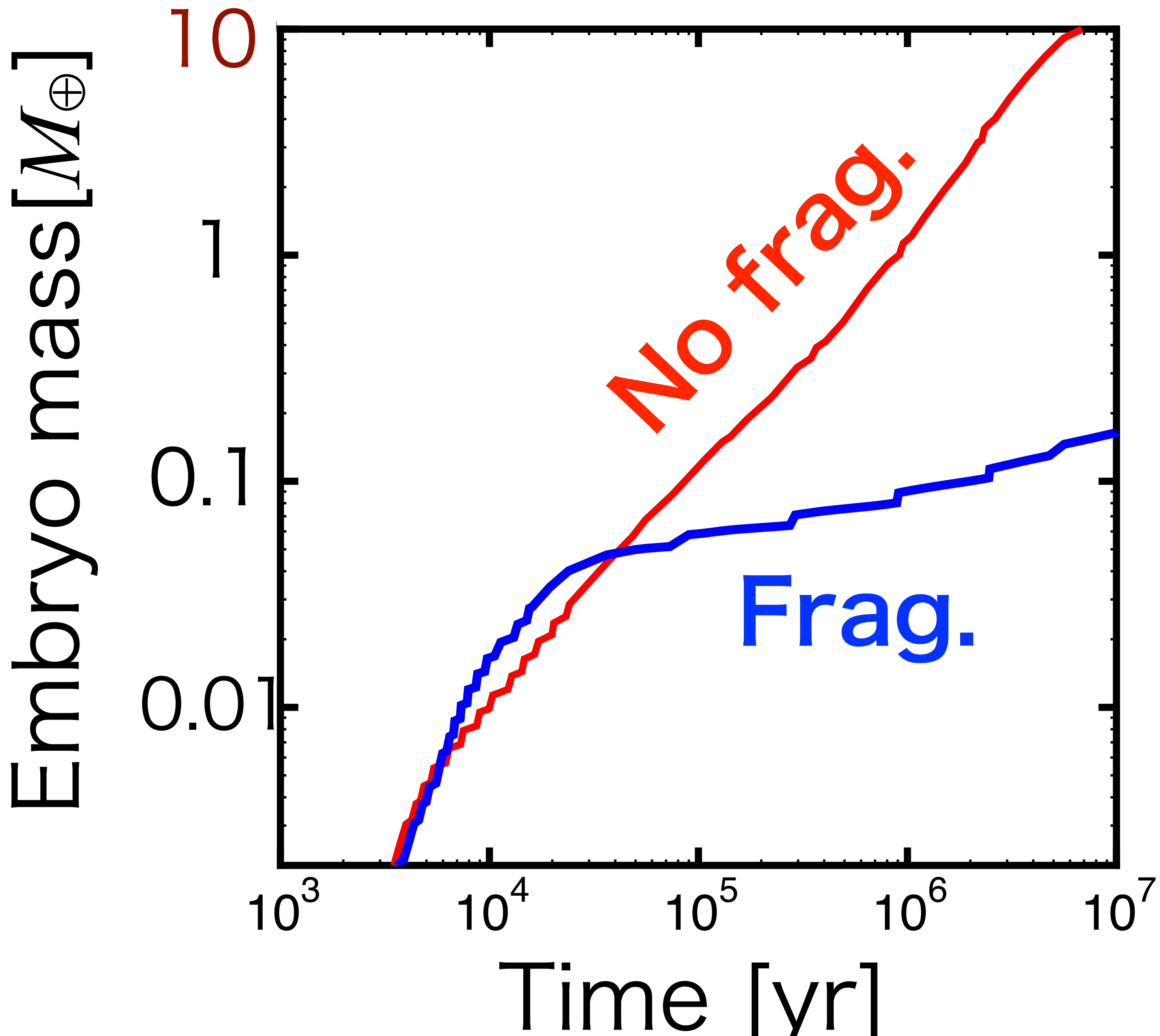


3xMMSN

Initial planetesimal:
1km

at 3.2AU

Embryo Growth



3xMMSN

Initial planetesimal: 1km

at 3.2AU

Fragmentation controls embryo growth.

Collision Cascade

$\bar{m}_{\mathrm{e}} \propto m \quad n(m) \propto m^{-\alpha}$

$$\int m_{\mathrm{e}}\, n(m)\Delta m\, \sigma v\, n(m')dm' \propto m^{11/3-2\alpha} (\bar{m}'/m)^{1-\alpha}$$

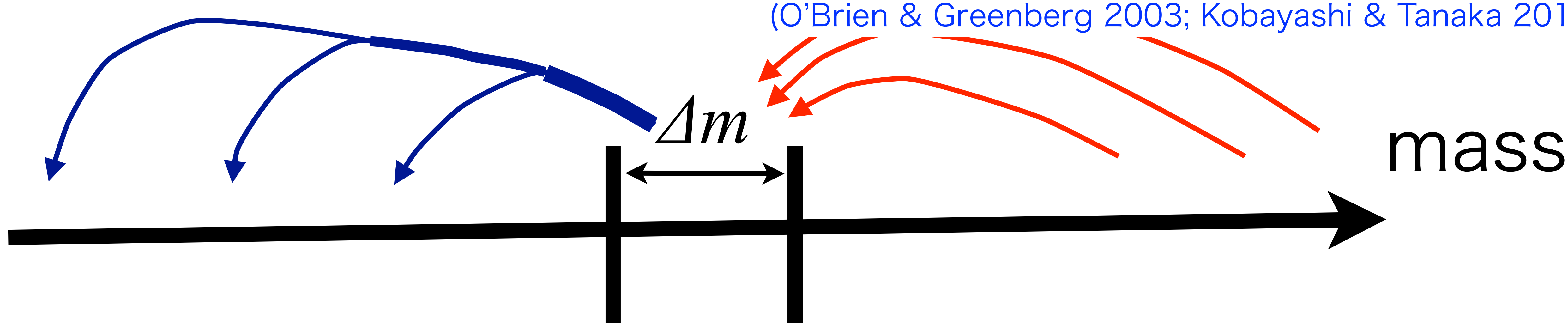
# of collisions

α=11/6 ($r^{-3.5}$)

(Dohnanyi 1969; Tanaka et al. 1996)

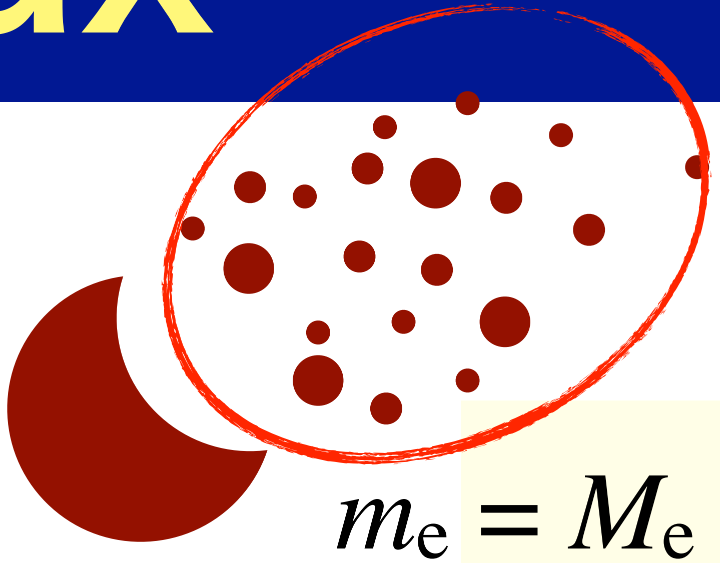
$\bar{m}'/m \propto m^{p} \quad \alpha = (11+3p)/(6+3p)$

(O'Brien & Greenberg 2003; Kobayashi & Tanaka 2010)



- Power-law mass distribution (System mass in large bodies).
- The mass flux is determined by m_e(m'/m).

Mass Flux



$m_e = M_e\, m_t$

Mass conservation:

$$\frac{\partial m n_s(m)}{\partial t} + \frac{\partial F(m)}{\partial m} = 0$$

$n_s(m)$: differential surface number density
$F(m)$: mass flux along the mass coordinate

For $n_s(m) = A\, m^{-\alpha}$,

$$\phi \approx \frac{m_p v^2}{2 m_t Q_D^*}$$

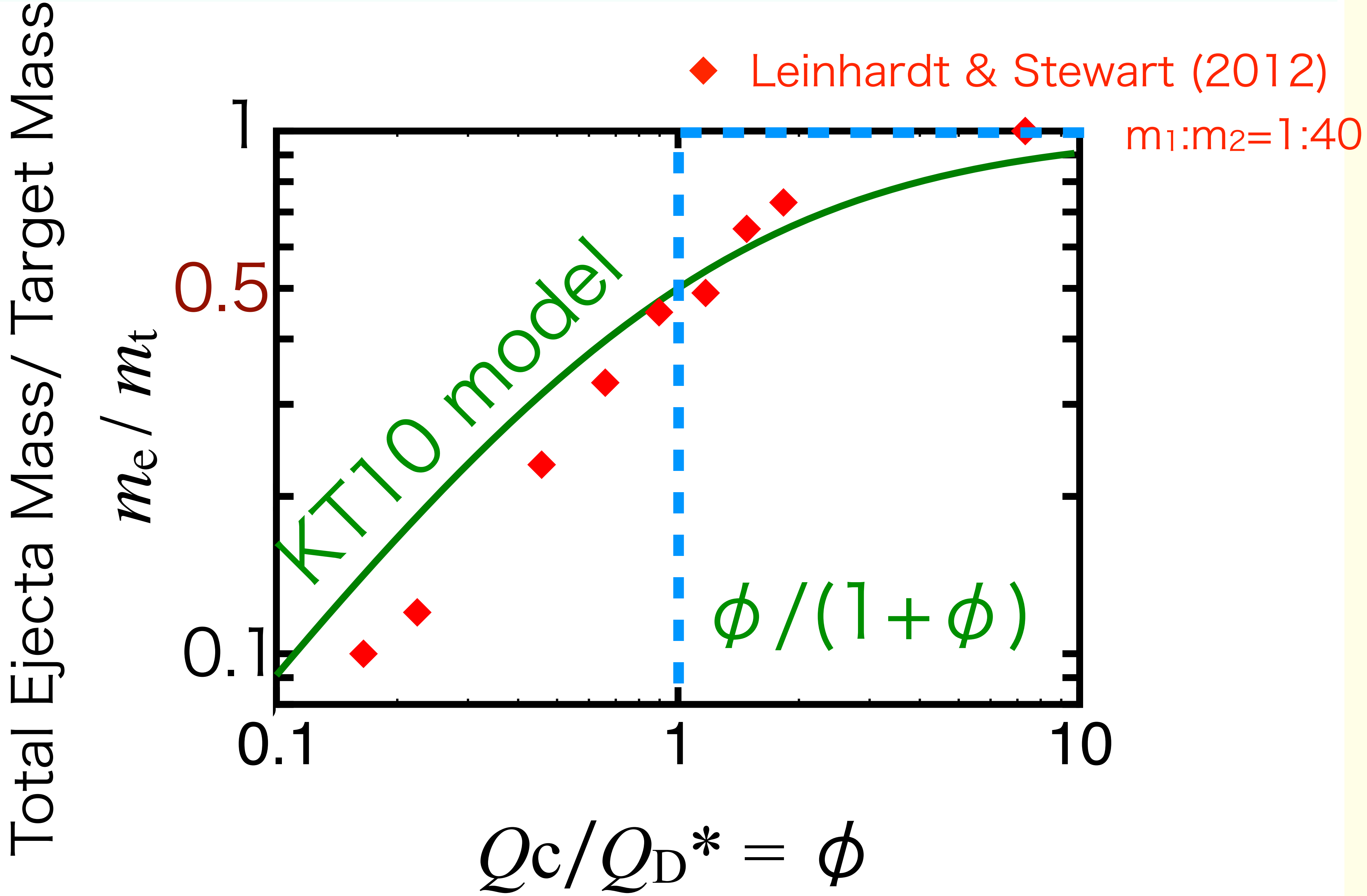
$$F(m) = -A^2 \Omega_K h_0 m^{\frac{11}{3} - 2\alpha} \left(\frac{v(m)^2}{2 Q_D^\star(m)} \right)^{\alpha - 1} \int_0^\infty d\phi \phi^{-\alpha}$$

$$\times \{ M_e(\phi) \ln M_m(\phi) - [1 - M_e(\phi)] \ln [1 - M_e(\phi)] \}$$

M_e: total ejecta mass scaled by target mass.

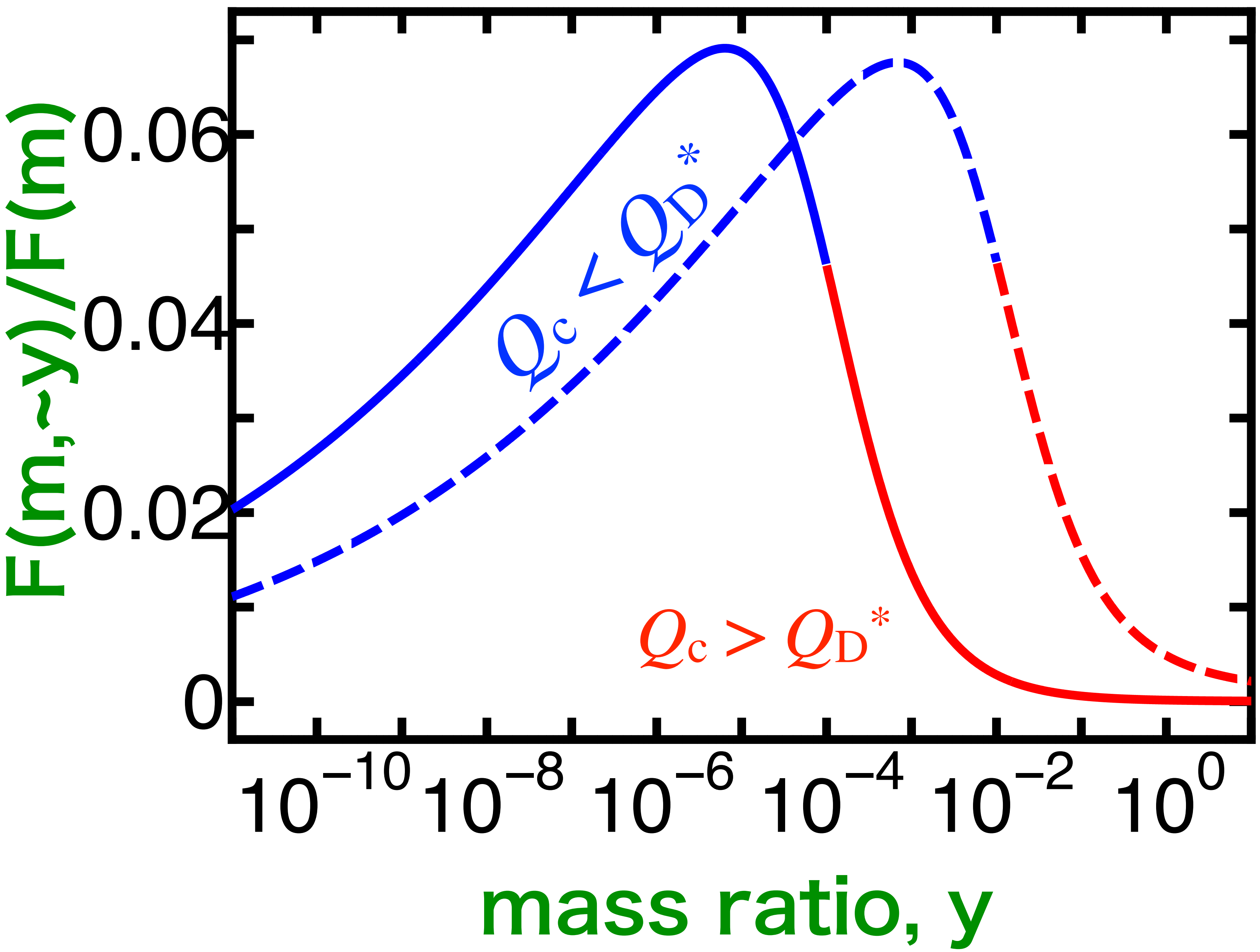
M_m: mean fragment mass divided by target mass

Fragmentation Model



Important mass ratio

$$\frac{v^2}{Q_\mathrm{D}^*} = 200, 2000$$



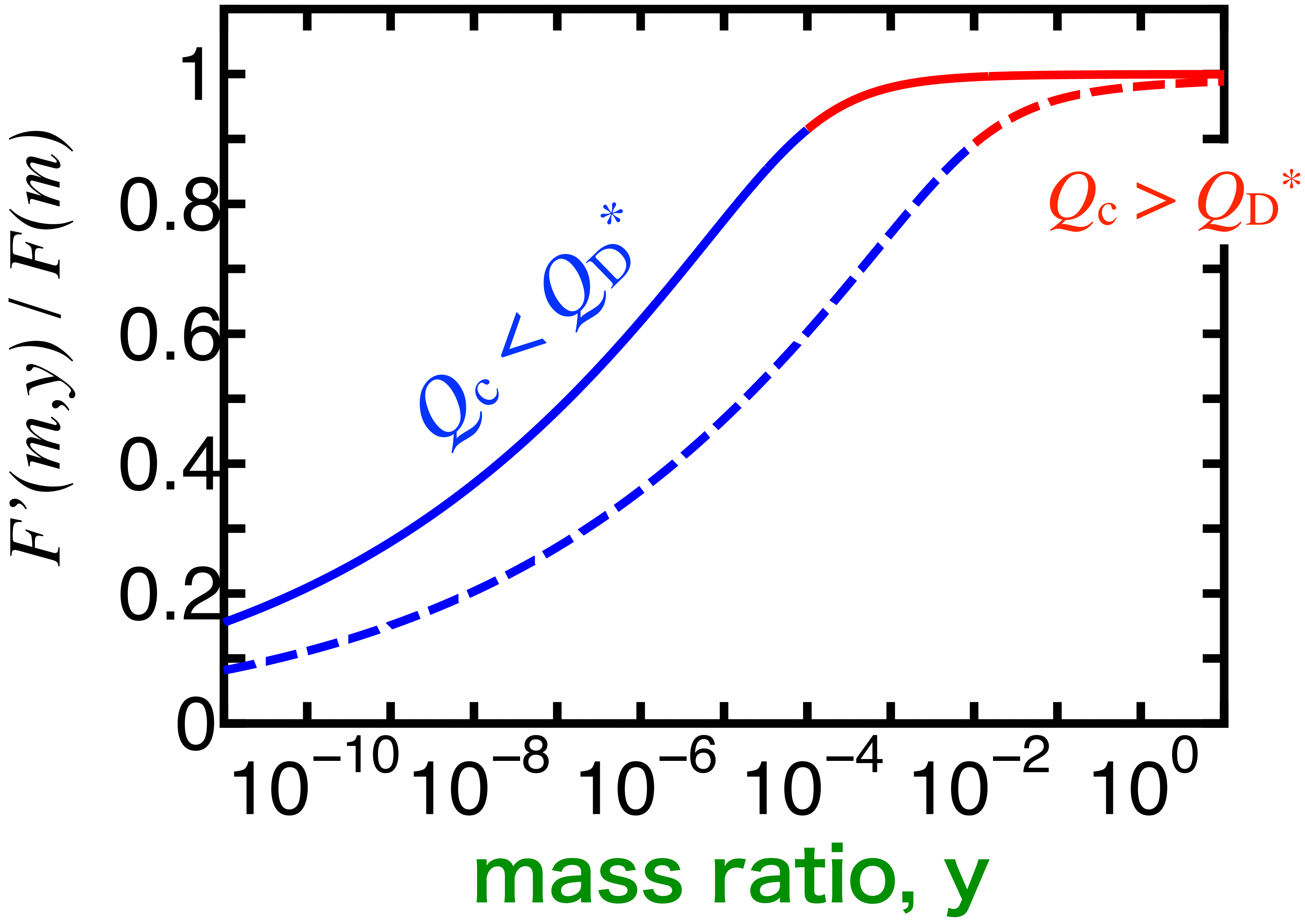
- Collisions with small bodies determine the flux.
- Erosive collision important.

$$\frac{\bar{m}'}{m} = 10^{-2}\text{–}10^{-1} \ \frac{Q_\mathrm{D}^*}{v^2}$$

$$\bar{m}_\mathrm{e} = 10^{-2}\text{–}10^{-1} \ m$$

Efficiency of erosive collisions

$$\frac{v^2}{Q_D^*} = 200, 2000$$



- Erosive collisions are 10 times more important for constant v^2/Q_D^*.
- For gravity regime, the efficiency is a factor 4-5.

Other Effects

- The power-law index does not change by erosive collisions.
- Previous studies show that the power-law distribution we analytically derived is modulated by a wavy structure.
- However, erosive collisions make the wavy structure smooth.

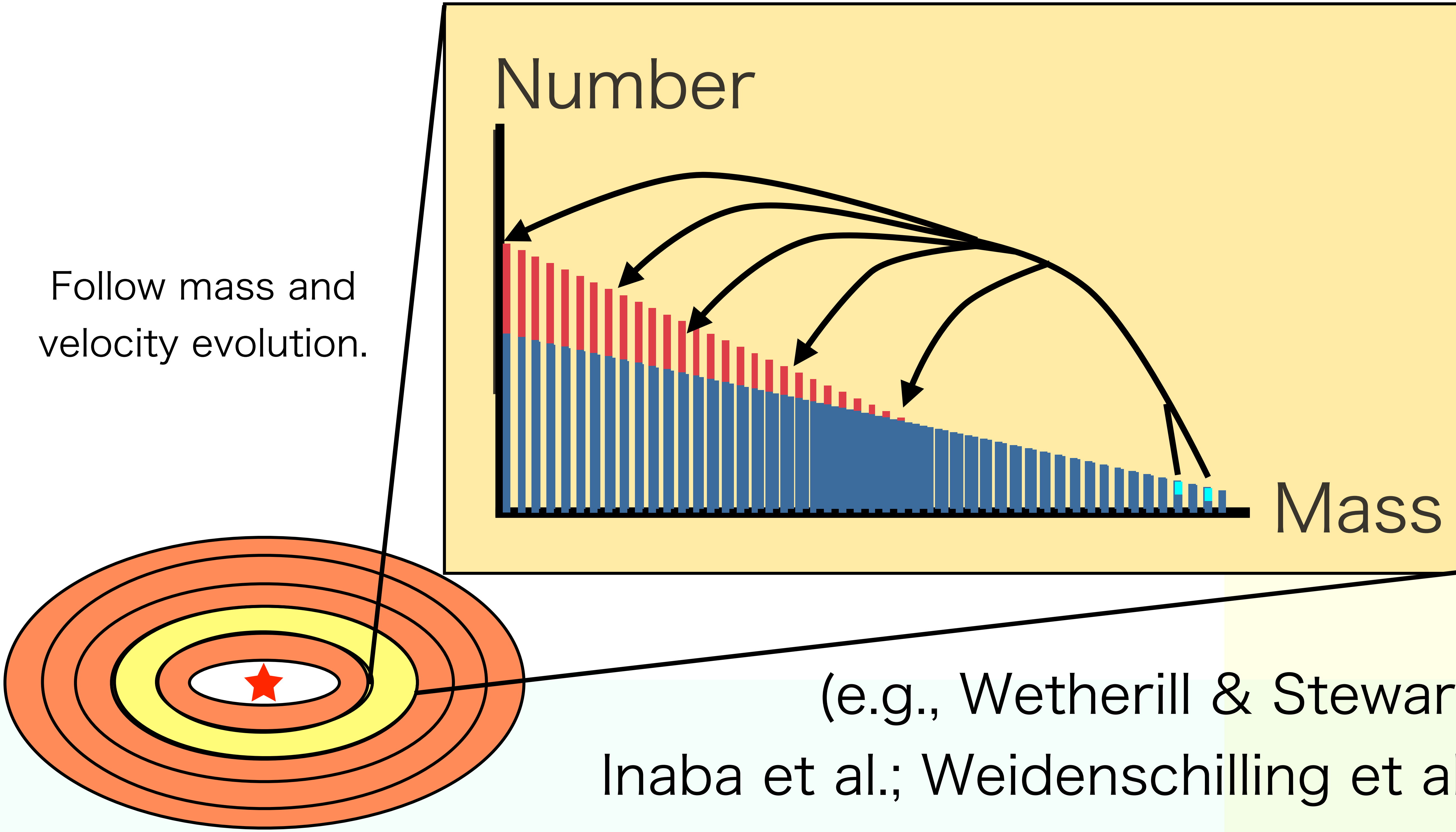
Erosive collisions also change the mass distribution of bodies in collisional cascade.

Planet Formation

- Runaway growth produces single planetary embryo in each annulus, while the total mass of leftover planetesimals surrounding the embryo is much more massive.
- The embryo grow through collisional accretion with planetesimals, until planetesimals deplete by collisional cascade.

Simulation

Follow mass and velocity evolution.



(e.g., Wetherill & Stewart; Inaba et al.; Weidenschilling et al.; Kenyon & Bromley)

Basic Equations

(see HK, Tanaka, Krivov, Inaba 2010)

$$
\begin{aligned}
\frac{\partial m n_{\rm s}(m,a)}{\partial t} = & \frac{m}{2}\Omega_{\rm K}\int_0^m dm_1 \int_{m-m_1-m_{\rm e}}^{\infty} dm_2 \\
& \times (h_{m_1,m_2}a)^2 n_{\rm s}(m_1,a) n_{\rm s}(m_2,a)\langle P_{\rm col}\rangle \\
& \times \delta(m-m_1-m_2+m_{\rm e}) \\
& -\Omega_{\rm K} m n_{\rm s}(m)\int_0^{\infty} dm_2 (h_{m,m_2}a)^2 n_{\rm s}(m_2,a)\langle P_{\rm col}\rangle \\
& +\frac{\partial}{\partial m}\Omega_{\rm K}\int_m^{\infty} dm_1 \int_0^{m_1} dm_2 (m_1+m_2) f(m,m_1,m_2) \\
& \times n_{\rm s}(m_1,a) n_{\rm s}(m_2,a)(h_{m_1,m_2}a)^2\langle P_{\rm col}\rangle \\
& -\frac{1}{a}\frac{\partial}{\partial a}[a m n_{\rm s}(m,a) v_{\rm drift}(m,a)],
\end{aligned}
$$

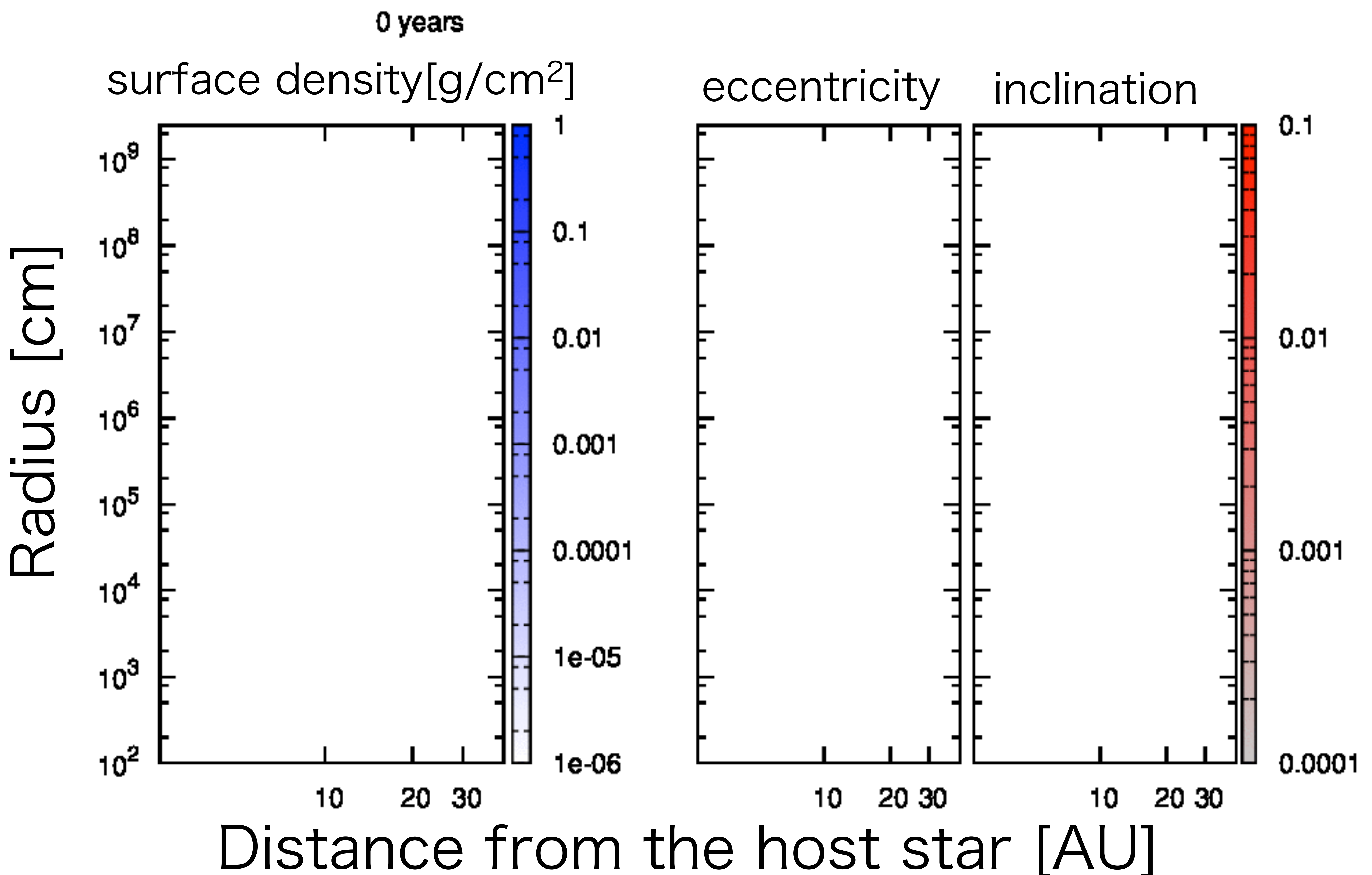
$$\frac{m_{\rm e}}{m_1+m_2} = \frac{\phi}{1+\phi}$$

$$\phi = m_1 m_2 v^2/2(m_1+m_2)^2 Q_{\rm D}^*$$

$$
(m_1+m_2) f(m,m_1,m_2) = \begin{cases} m_{\rm e}\left(\dfrac{m}{m_{\rm L}}\right)^{-b} & \text{for} \quad m < m_{\rm L}, \\ m_{\rm e} & \text{for} \quad m \geq m_{\rm L}, \end{cases}
$$

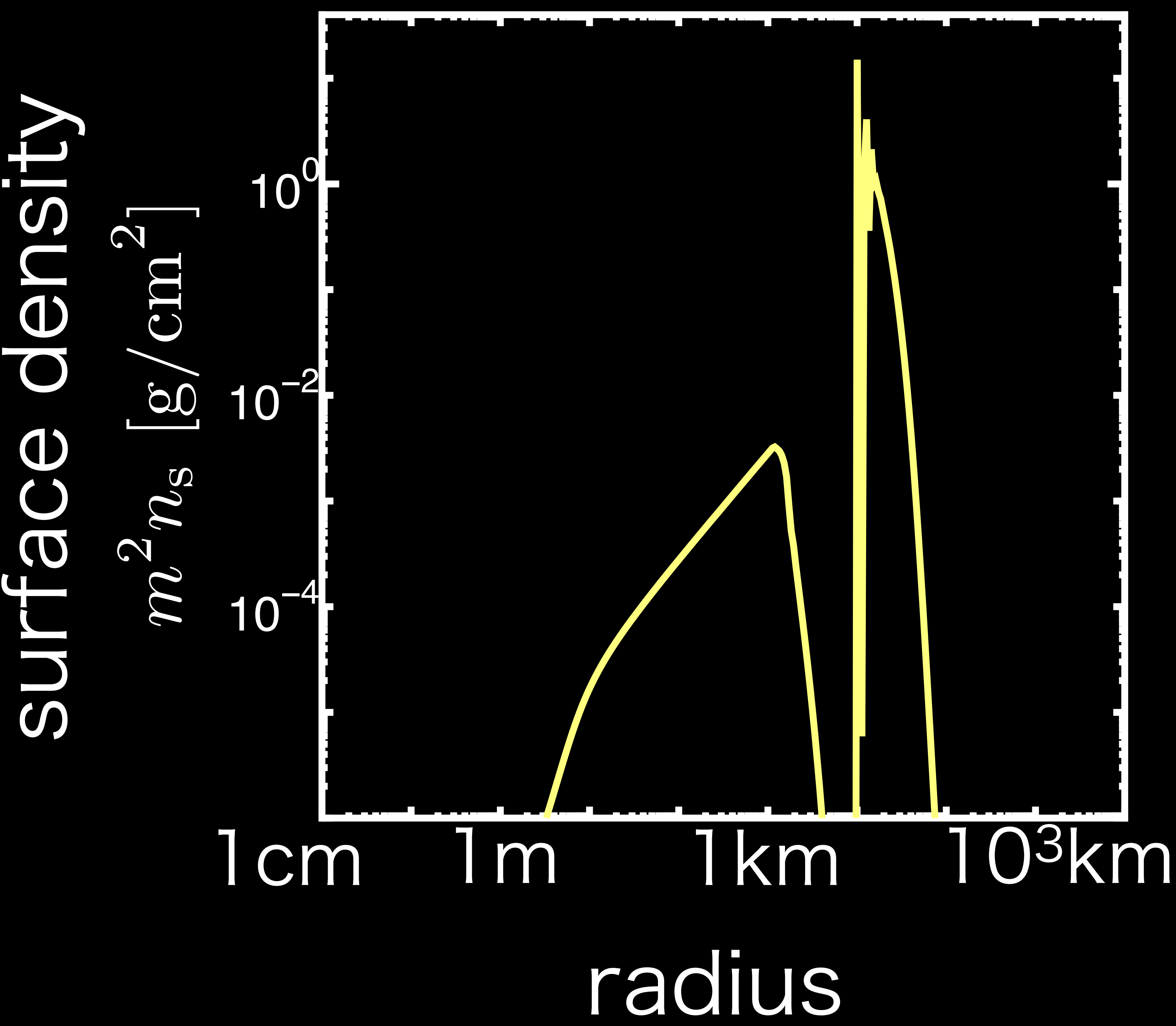
$$
\begin{aligned}
\frac{de^{*2}}{dt} &= \left(\frac{de^{*2}}{dt}\right)_{\rm grav} + \left(\frac{de^{*2}}{dt}\right)_{\rm gas} + \left(\frac{de^{*2}}{dt}\right)_{\rm coll}, \\
\frac{di^{*2}}{dt} &= \left(\frac{di^{*2}}{dt}\right)_{\rm grav} + \left(\frac{di^{*2}}{dt}\right)_{\rm gas} + \left(\frac{di^{*2}}{dt}\right)_{\rm coll},
\end{aligned}
$$

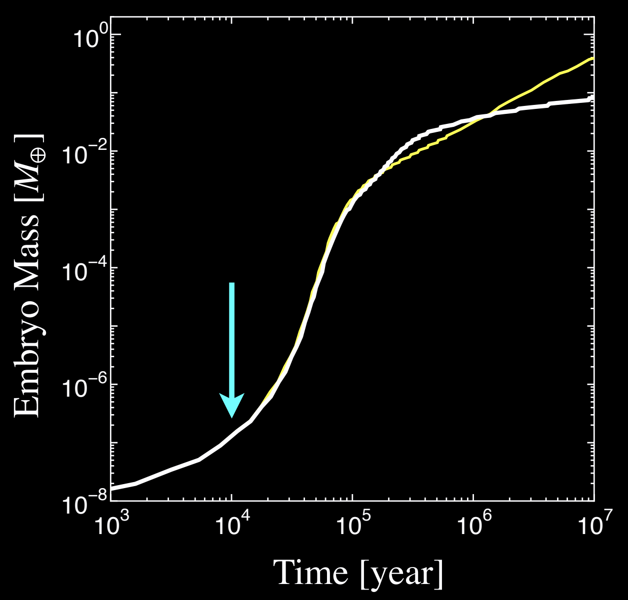
Mass & velocity evol.



size evolution

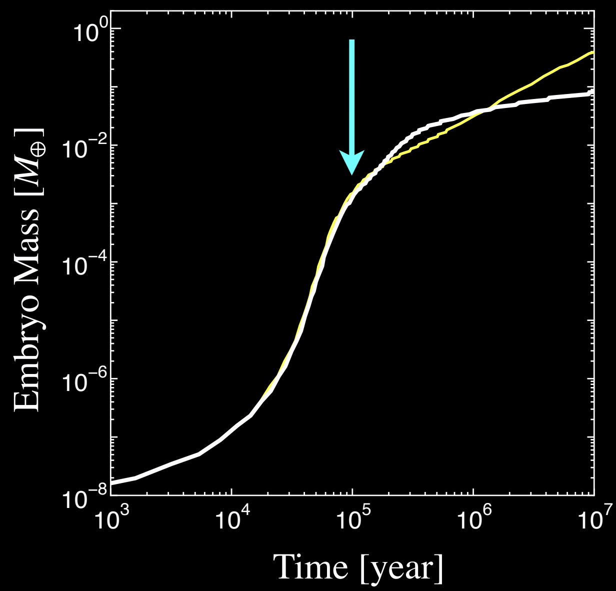
10^4 years



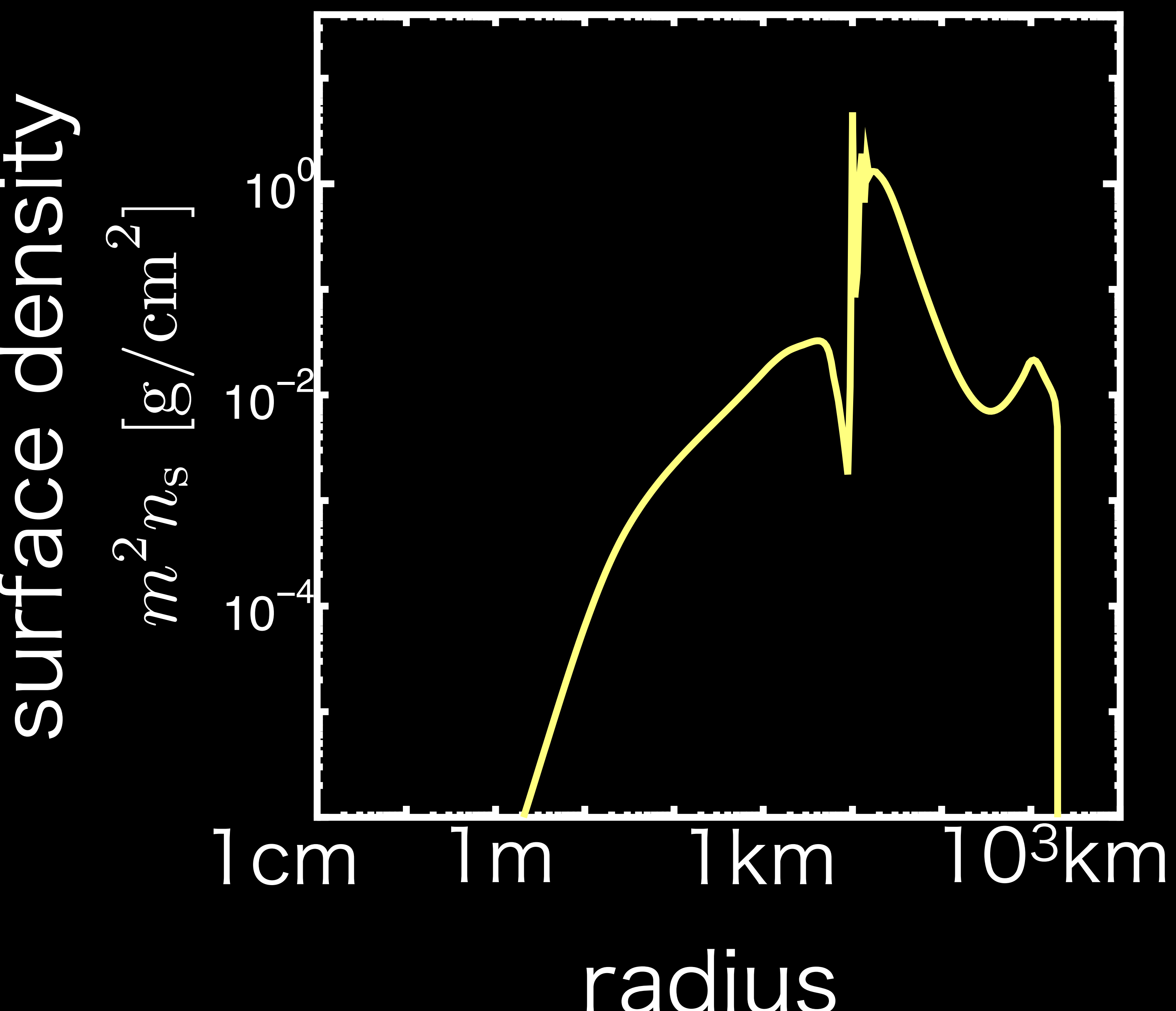


3.2AU

10^5 年後



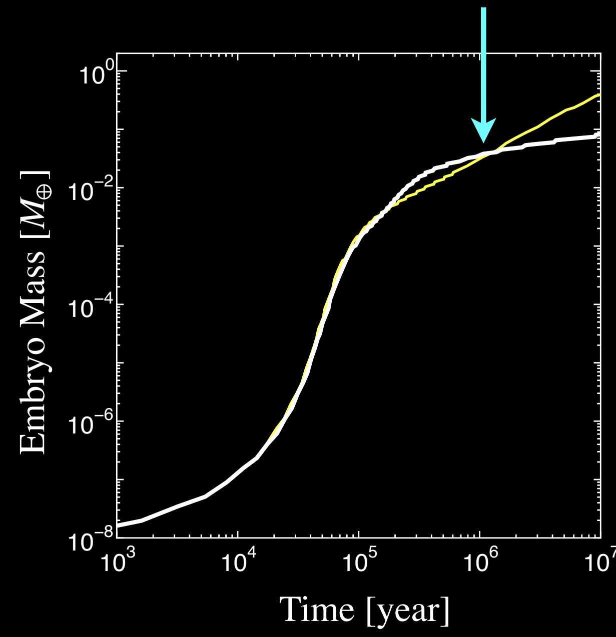
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^3
10^4
10^5
10^6
10^7
Embryo Mass $[M_\oplus]$
Time [year]

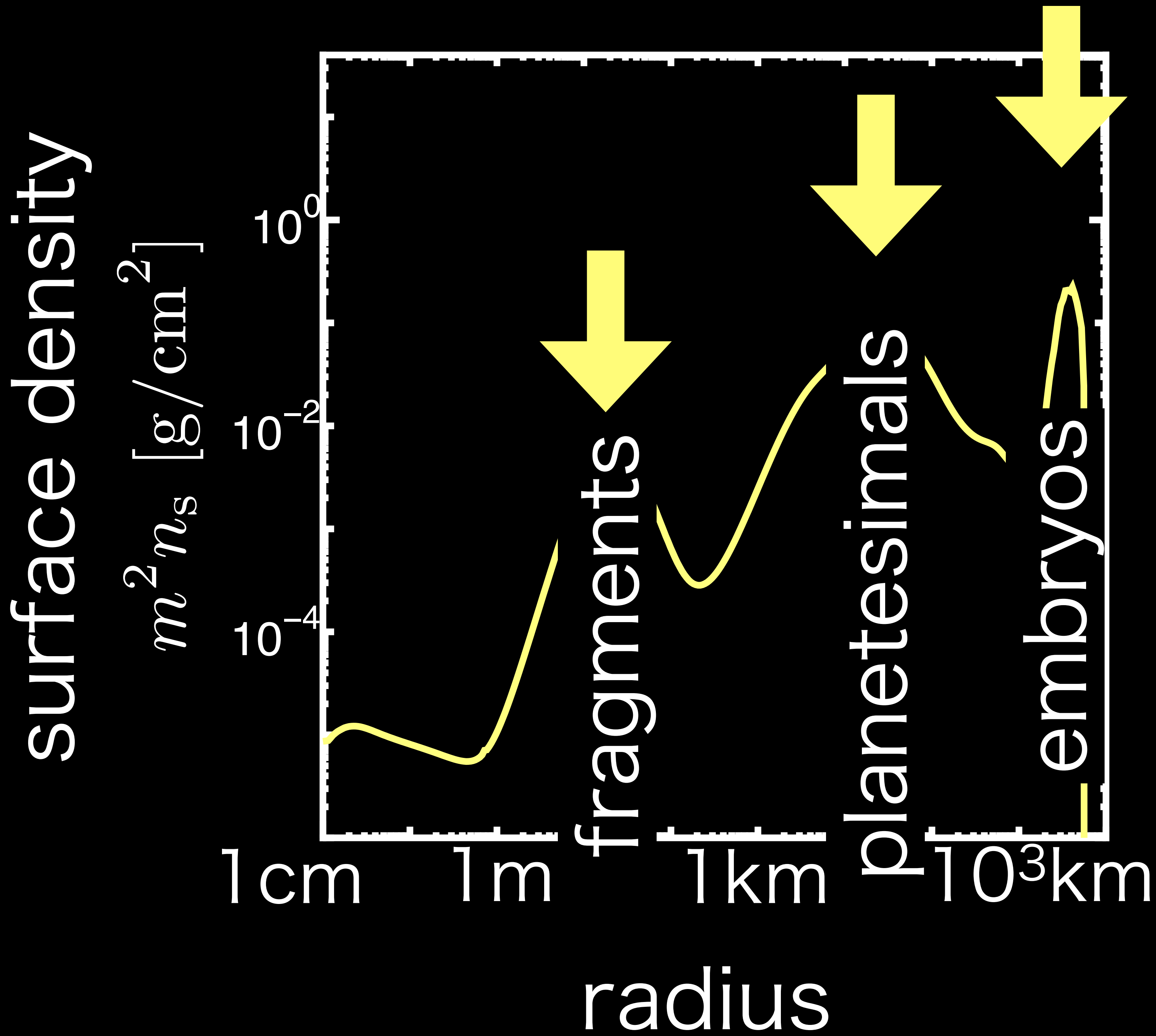


surface density
$m^2 n_s$ $[\mathrm{g/cm^2}]$
10^0
10^{-2}
10^{-4}
1cm
1m
1km
10^3km
radius

3.2AU

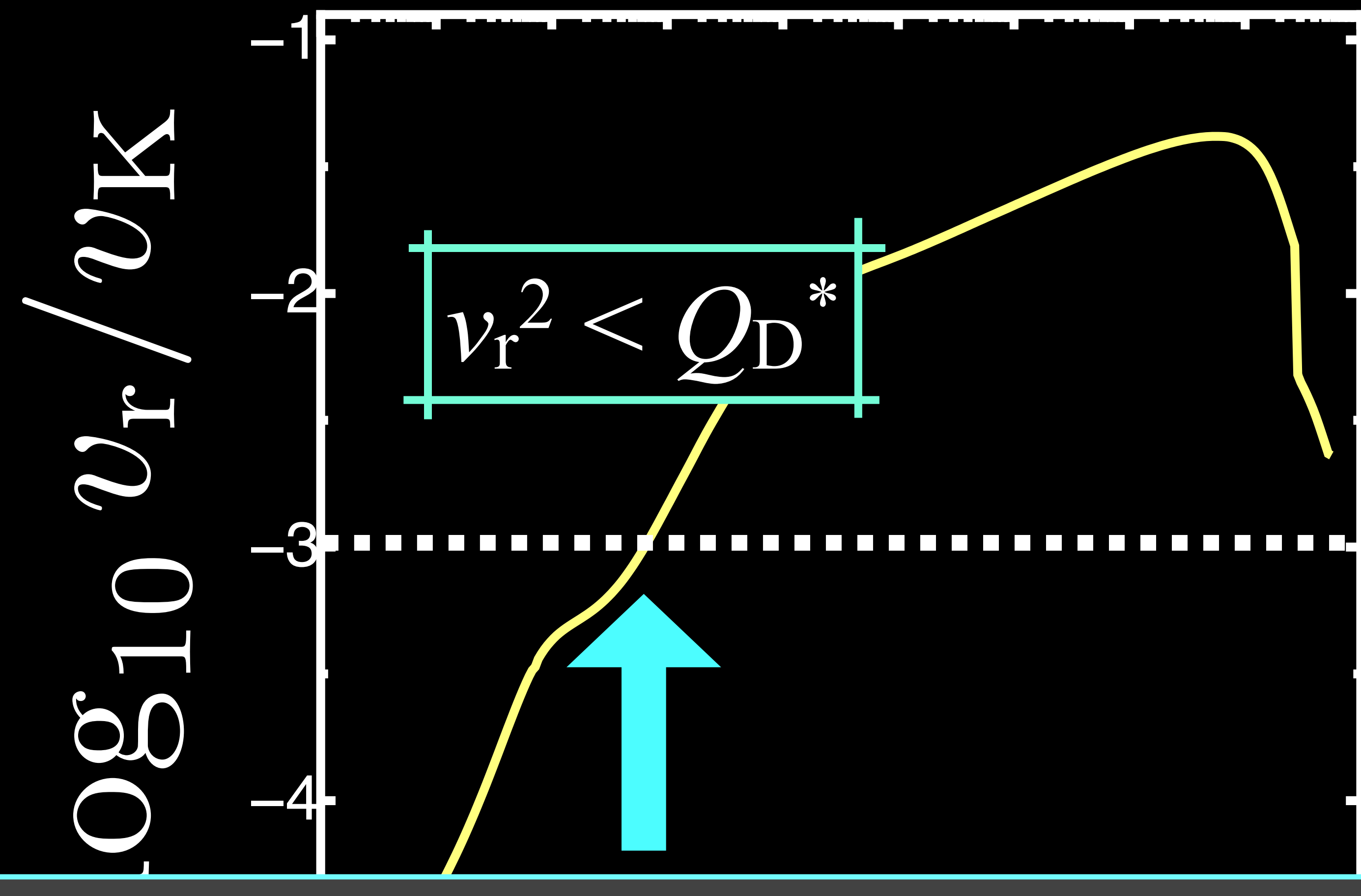
10^6 years





3.2AU

Fragment size



1x10⁶yr

$$4.7 \left(\frac{M}{0.1 M_\oplus}\right)^{-\frac{1}{2}} \left(\frac{a}{3.2\,\mathrm{AU}}\right)^{\frac{1}{8}} \left(\frac{Q_D^\star}{3\times10^6 \mathrm{erg/g}}\right) \mathrm{m}$$

U

radius

Final Mass

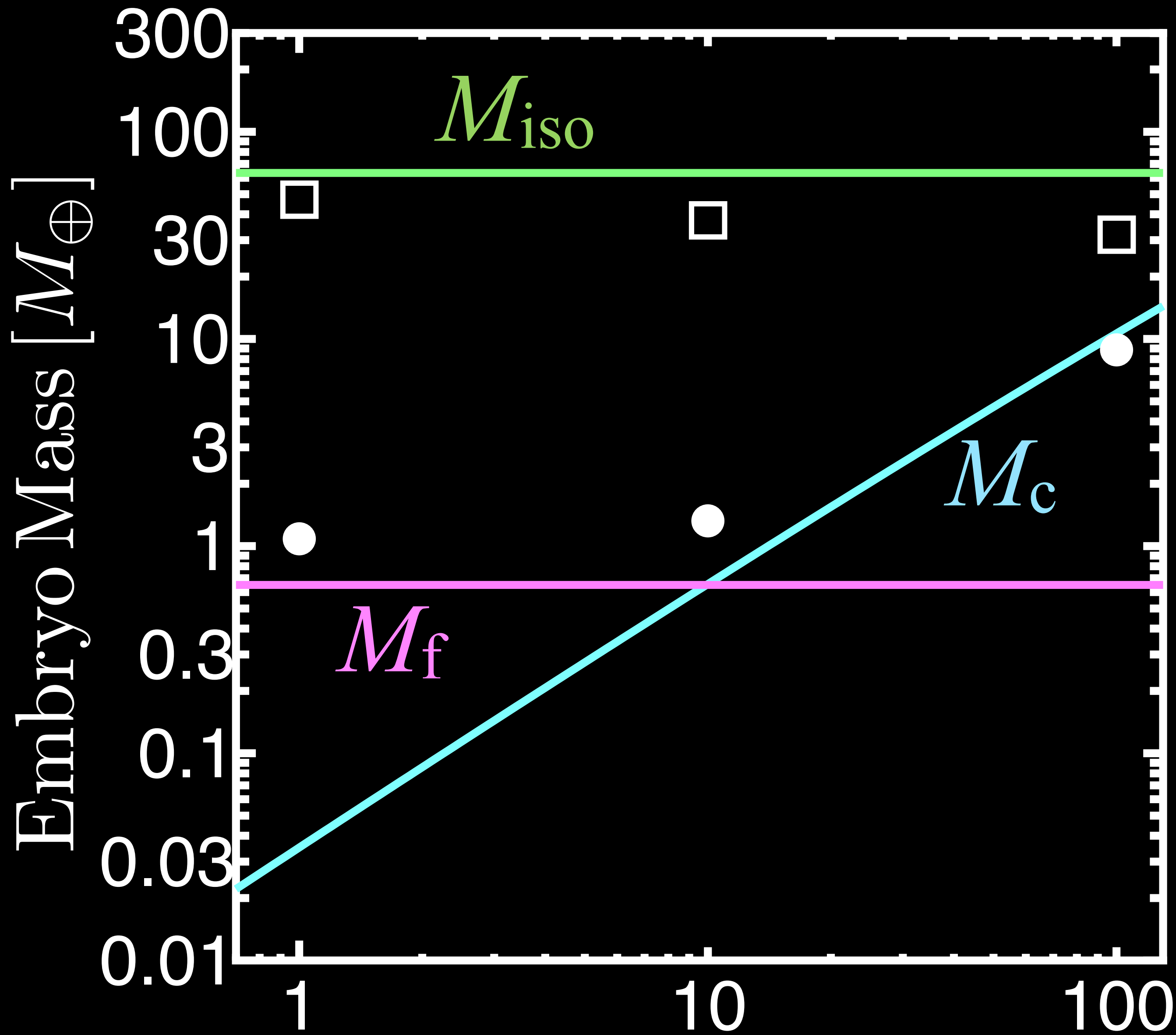
10 x MMSN

10Myr

Fragmentation

● with

□ no



initial planetesimal radius [km]

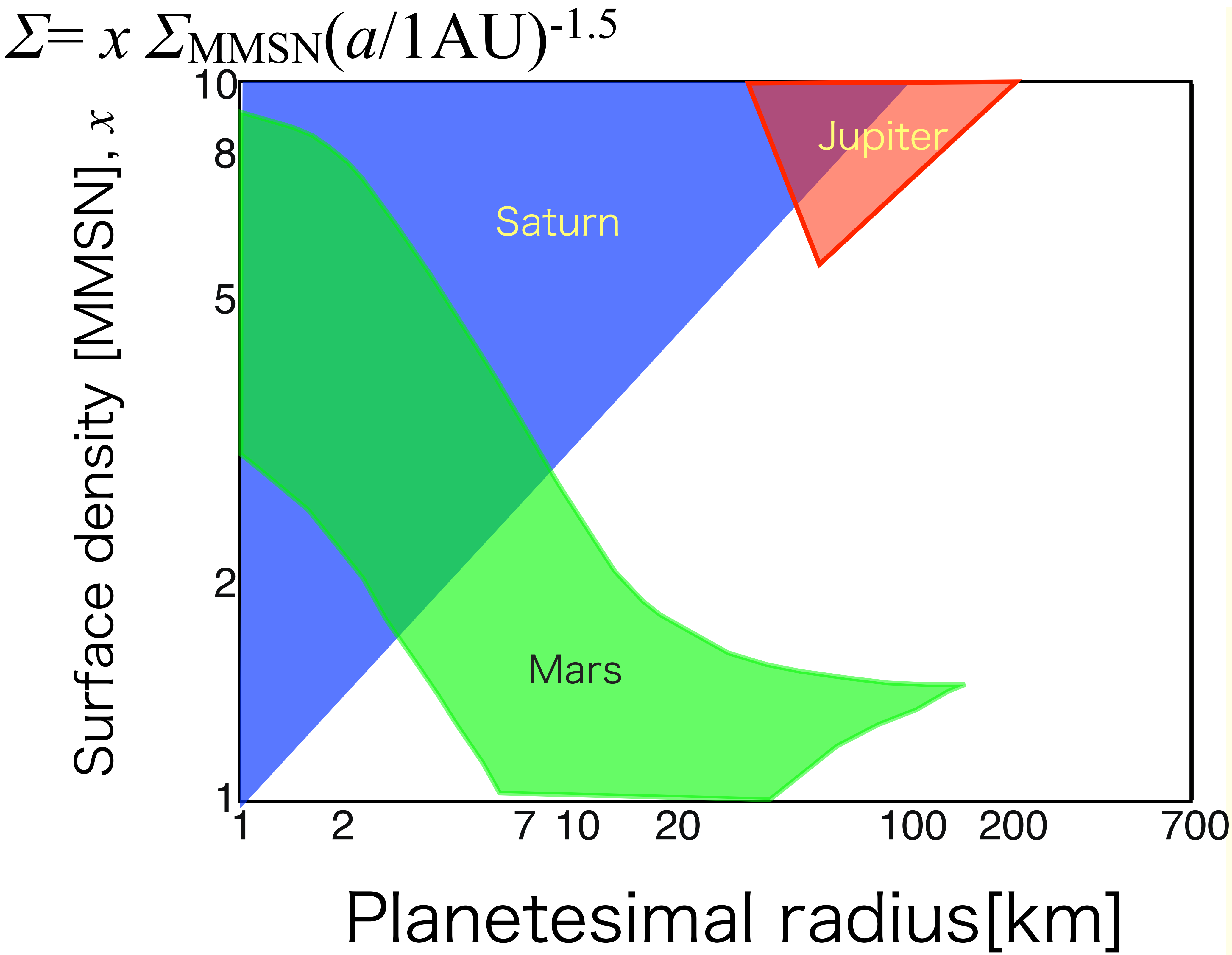
3.2AU

(Kobayashi et al. 2010; 2011)

Summery

- Collisional cascade:
  - The total ejecta mass from a single collision is most important and the others are negligible.
  - Erosive collisions mainly determine the mass flux and the depletion time of bodies.
- Planet formation:
  - Massive embryos are formed in a massive disk.
  - Large planetesimals produce massive embryos but via slow growth (and vice versa).

Application: Solar System



- Jupiter
  - Kobayashi+11
- Saturn
  - Kobayashi+12
- Mars
  - Kobayashi& Dauphas13

Solar System Formation

まとめ